Spans of lenses

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Bx, Athens, March 28, 2014

A wise person…

"For a short talk, use only evocative pictures"

(no words, except possibly in titles)

But. . .

Prepare to be disappointed

Picture 1

$$X \longrightarrow Y$$

$$g : X \longrightarrow Y$$

$$X \longleftarrow Y \times X : p$$

Asymmetric lens from X to Y

$$X \longrightarrow Y$$

$$g : X \longrightarrow Y$$

$$X \longleftarrow Y \times X : p$$

Classical, set-based, asymmetric lens from X to Y

$$X \longrightarrow Y$$

$$g : X \longrightarrow Y$$

$$X \longleftarrow Y \times X : p$$

Classical, set-based, asymmetric lens from X to Y

$$X \longrightarrow Y$$

$$g : X \longrightarrow Y$$

$$X \longleftarrow Y \times X : p$$

(cf c-lens, d-lens, e-lens, q-lens, r-lens, v-lens, w-lens . . .)

Picture 2

$$X \longrightarrow Y$$

$$R : X \times C \longrightarrow Y \times C$$

$$X \times C \longleftarrow Y \times C : L$$

Symmetric lens from X to Y

$$X \longrightarrow Y$$

$$R : X \times C \longrightarrow Y \times C$$

$$X \times C \longleftarrow Y \times C : L$$

RL-lens from X to Y

$$X \longrightarrow Y$$

$$R : X \times C \longrightarrow Y \times C$$
$$X \times C \longleftarrow Y \times C : L$$

Two RL-lenses from X to Y

$$X \longrightarrow Y$$

$$R : X \times C \longrightarrow Y \times C$$

$$X \times C \longleftarrow Y \times C : L$$

$$R' : X \times C' \longrightarrow Y \times C'$$

$$X \times C' \longleftarrow Y \times C' : L'$$

Two RL-lenses from X to Y

$$X \longrightarrow Y$$

$$R : X \times C \longrightarrow Y \times C$$

$$X \times C \longleftarrow Y \times C : L$$

$$r \subseteq C \times C'$$

$$R' : X \times C' \longrightarrow Y \times C'$$

$$X \times C' \longleftarrow Y \times C' : L'$$

Two RL-lenses from X to Y

$$X \longrightarrow Y$$

$$R : X \times C \longrightarrow Y \times C$$

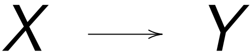
$$X \times C \longleftarrow Y \times C : L$$

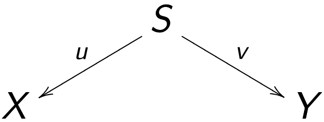
$$r \subseteq C \times C' \qquad \equiv_{HPW}$$

$$R' : X \times C' \longrightarrow Y \times C'$$

$$X \times C' \longleftarrow Y \times C' : L'$$

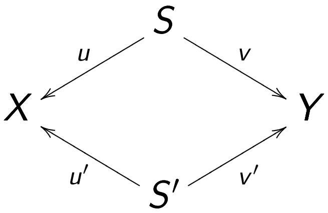
A span of (asym) lenses from X to Y





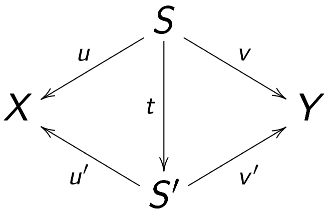
Two spans of (asym) lenses from X to Y

$$X \longrightarrow Y$$



Equivalent spans of (asym) lenses from X to Y

$$X \longrightarrow Y$$



Take home messages (1)

- A lens is a generalization of a bijection
- So “up to isomorphism” becomes “up to lens-equivalence”
- Symmetric lenses are constructed by taking spans of asymmetric lenses up to lens-equivalence

RL-equivalent RL-lenses from X to Y

$$X \longrightarrow Y$$

$$R : X \times C \longrightarrow Y \times C$$

$$X \times C \longleftarrow Y \times C : L$$

$$\text{Lens } t : C \longrightarrow C' \qquad \equiv_{RL}$$

$$R' : X \times C' \longrightarrow Y \times C'$$

$$X \times C' \longleftarrow Y \times C' : L'$$

Take home messages

- A lens is a generalization of a bijection
- So “up to isomorphism” becomes “up to lens-equivalence”
- Symmetric lenses are constructed by taking spans of asymmetric lenses up to lens-equivalence
- RL-lenses need the right domains:
-     Consistent triples
-     James’ monads
-     Zinovy’s corrs

A wise person. . .

“For a short talk, use only evocative pictures”

but . . .

A wise mathematician. . .

“A talk isn’t a talk without a theorem”

so . . .

Relating RLs and spans

Theorem

The category of normalised RL-lenses modulo RL-equivalence

is equivalent to

The category of spans of asymmetric lenses modulo lens-span-equivalence.