Principles of Guarded Structural Indexing

On Guarded Simulations and Acyclic First Order Languages

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Background

- Increased interest in tree-based and graph-based data formats: XML, RDF, JSON, social networks

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Central Question

Can structural indexes be generalized for arbitrary relational databases?

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- We group nodes such that any query $Q \in \mathcal{Q}$ can be answered
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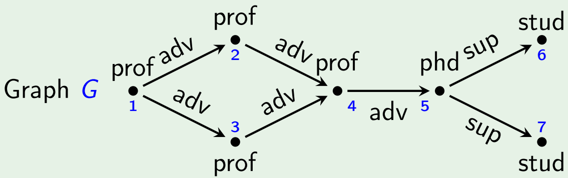
  - directly on G but using pruning information from the index.
- Since the index is typically (much) smaller than G itself, this can be significantly faster than evaluating Q directly over G.

Structural Indexes

Example

Example (Academic relations graph)



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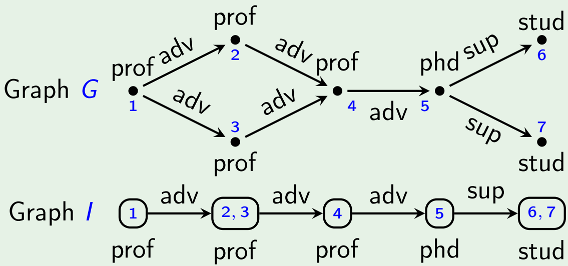

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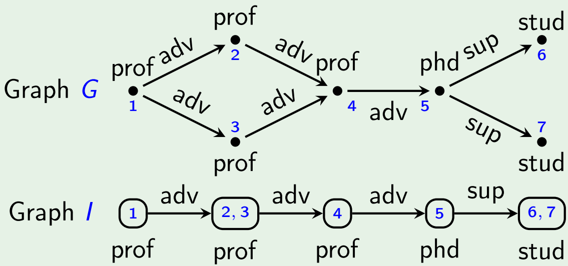

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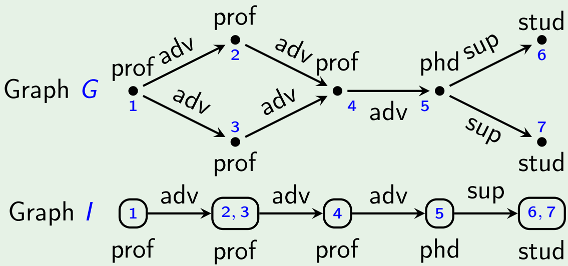

- Each node in I is actually a set of nodes in G.
- There is an edge between sets V and W in I if there is an edge between some $v \in V$ and some $w \in W$ in G.

(Bi)simulation on Labeled Graphs

Definition

A simulation of G_1 in G_2 is a binary relation $T \subseteq V_1 \times V_2$ s.t.

(lab) it relates only nodes with the same label, and

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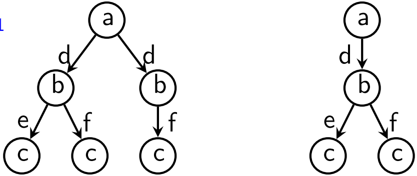

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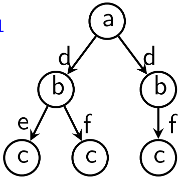

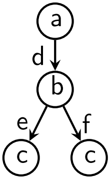

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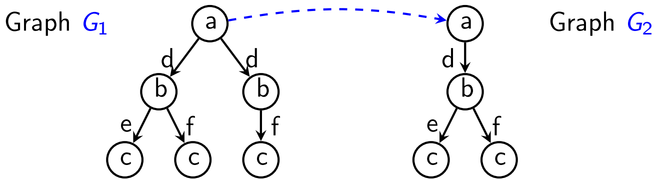

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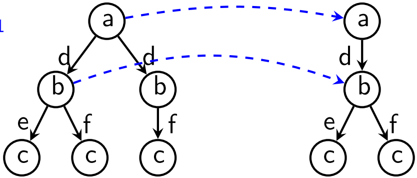

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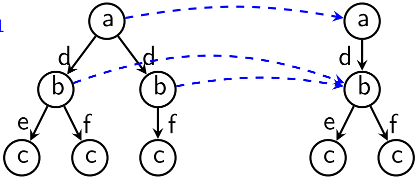

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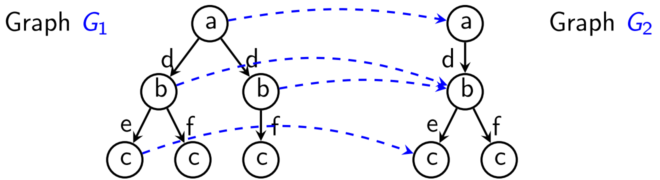

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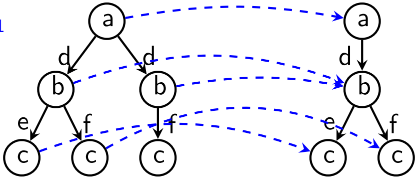

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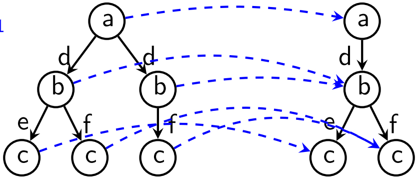

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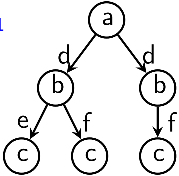

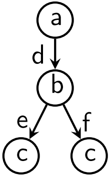

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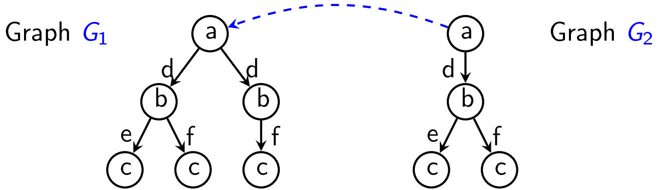

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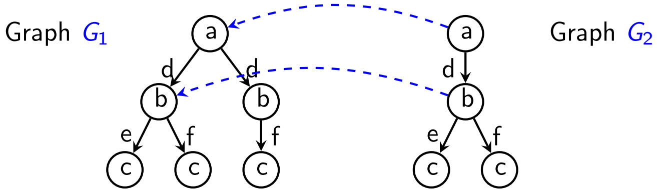

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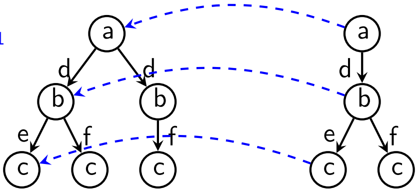

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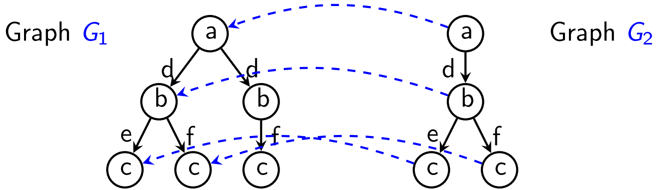

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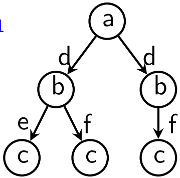

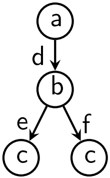

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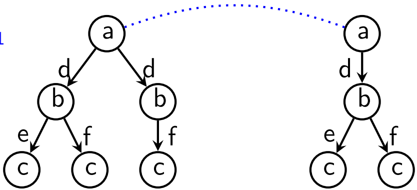

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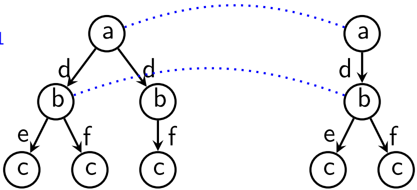

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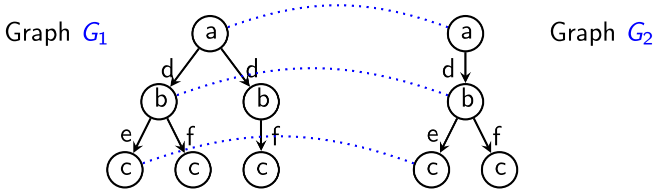

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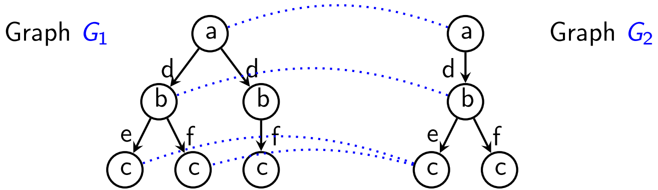

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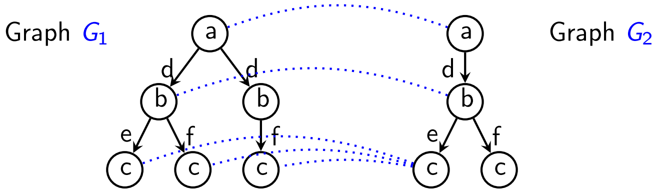

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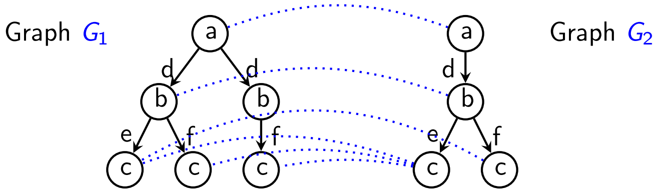

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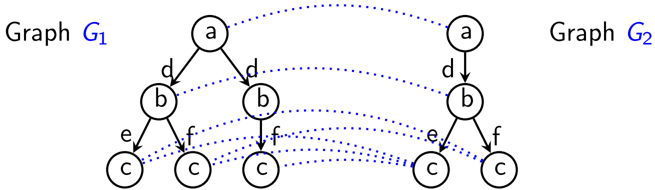

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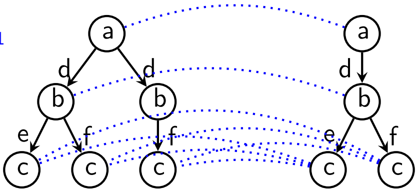

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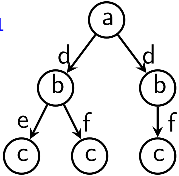

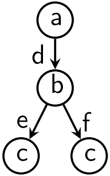

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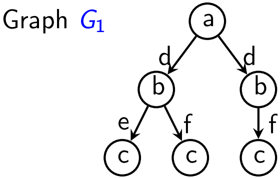

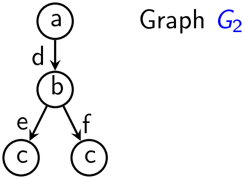

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(Bi)simulation on Labeled Graphs

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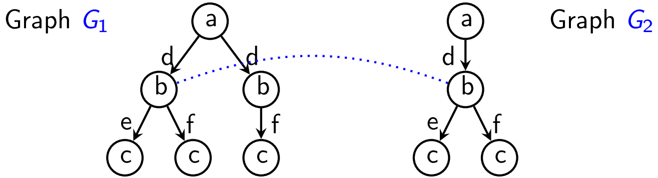

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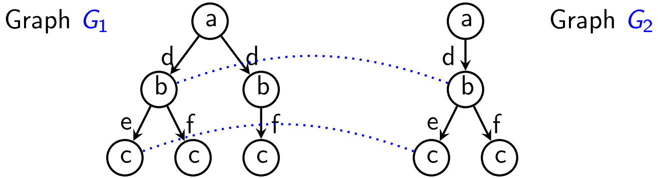

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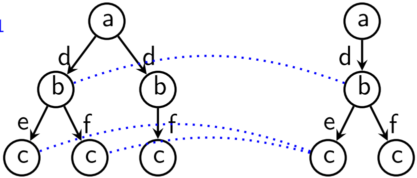

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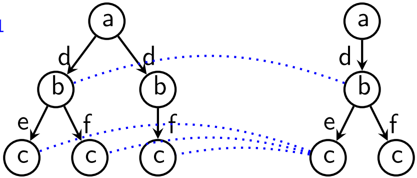

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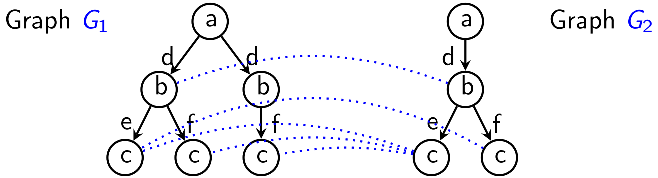

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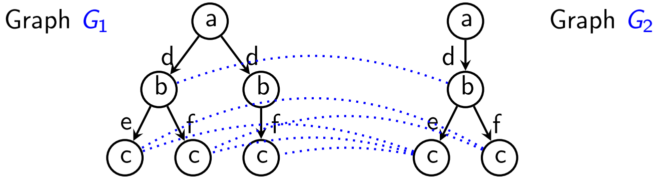

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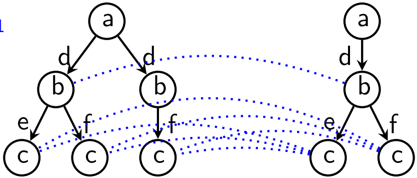

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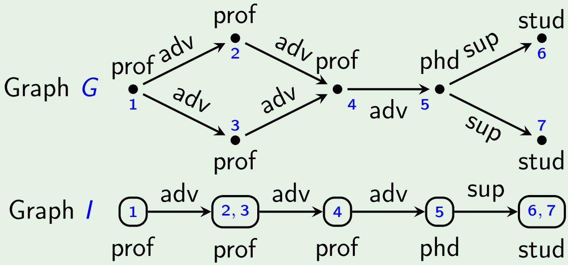
Simulation Relations for Structural Indexes

Example (Academic relations graph)



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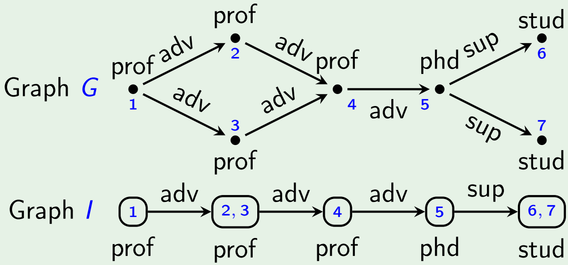

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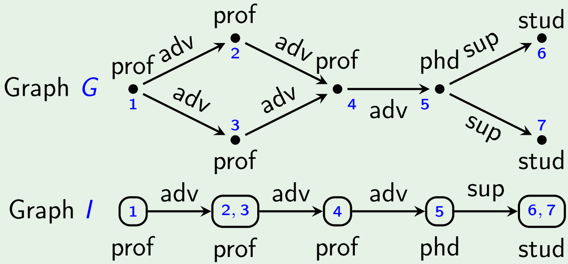

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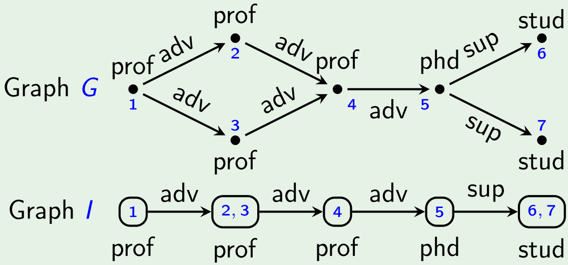

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  - Applying Q on I gives the node $\{2, 3\}$ which is the correct answer

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  - Formally: All variables in the head occur in a single atom in the body, e.g., $ans(b, c) \leftarrow R(a, b), S(b, c, d), R(b, d)$.
  - This keeps the indexes small

Indistinguishability under Conjunctive Queries

All conjunctive queries are invariant under homomorphisms:

Theorem ([Chandra & Harel, 1980])

For all databases db_1 and db_2 and all tuples $\overline{a}_1$ and $\overline{a}_2$, if there exists a homomorphism f from db_1 to db_2 such that $f(\overline{a}_1) = \overline{a}_2$, then for every conjunctive query Q, if $\overline{a}_1 \in Q(db_1)$ then also $\overline{a}_2 \in Q(db_2)$.

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Invariance under homomorphisms in fact is a characterization of the conjunctive queries (modulo union):

Theorem ([Rossman, 2008])

A query expressible in first order logic (FO) is invariant under homomorphisms on finite structures if, and only if, it is equivalent in the finite to a union of conjunctive queries.

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  - Start from tractable relations such as simulation and bisimulation.
- Main informal result: leads to the same answer.

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- But what are then the edges?

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Definition (Equality type)

For tuples $\overline{a} = (a_1, \ldots, a_k)$ and $\overline{b} = (b_1, \ldots, b_l)$ their equality type is $eqtp(\overline{a}, \overline{b}) := \{(i, j) \mid a_i = b_j\}$.

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s_1	$r(n, o, p)$
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$eqtp(t_1, t_2) = \{(1, 2)\}$ and $eqtp(t_1, t_1) = \{(1, 1), (2, 2), (3, 3)\}$.

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The (bi)simulation relations

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A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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D_1	
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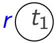

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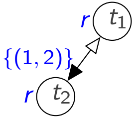

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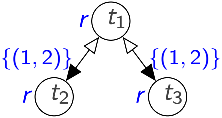
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$



D_2	

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

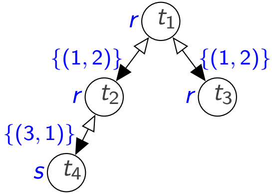
Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

(lab) it relates only facts with the same relation name, and

(forth) for every $(r(\overline{a}), r(\overline{b})) \in T$ and every $s(\overline{c}) \in D_1$ there exists $s(\overline{d}) \in D_2$ such that $(s(\overline{c}), s(\overline{d})) \in T$ and $eqtp(\overline{a}, \overline{c}) \subseteq eqtp(\overline{b}, \overline{d})$.

D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$



D_2	

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

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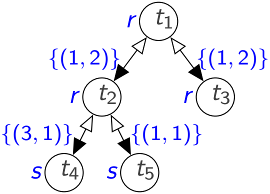
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$



D_2	

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

Definition

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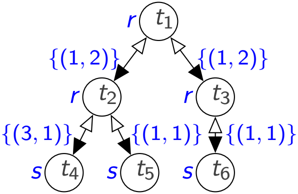
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

Definition

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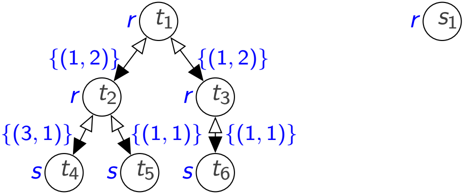
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	
s_1	$r(n, o, p)$

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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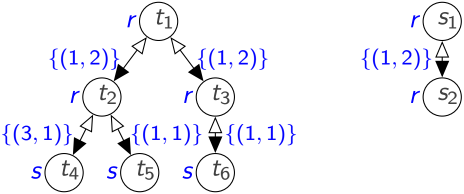
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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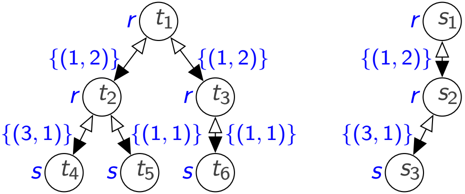
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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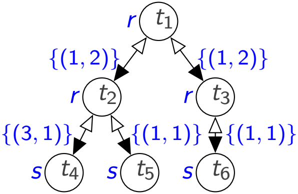
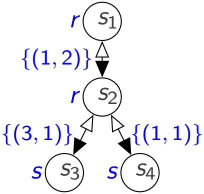
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$





D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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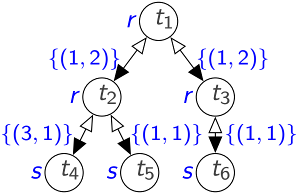
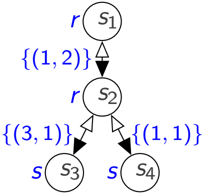
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$





D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

A guarded simulation of G_1 in G_2:

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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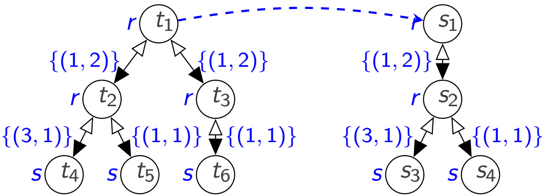
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

A guarded simulation of G_1 in G_2:

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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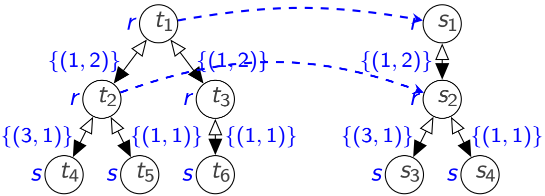
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

A guarded simulation of G_1 in G_2:

Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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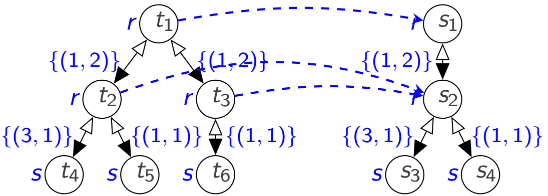
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

A guarded simulation of G_1 in G_2:

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A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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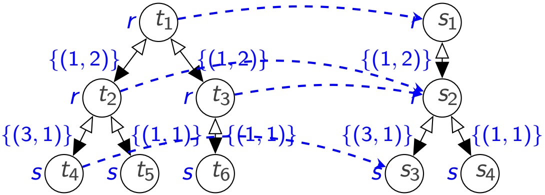
	D_1
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



	D_2
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

A guarded simulation of G_1 in G_2:

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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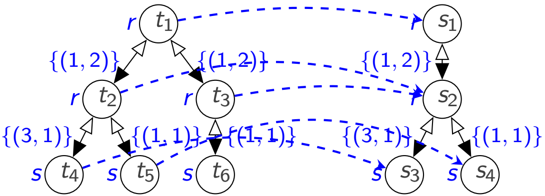
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

A guarded simulation of G_1 in G_2:

Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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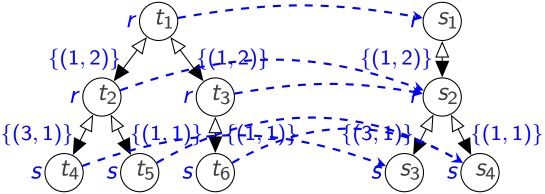
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

A guarded simulation of G_1 in G_2:

Guarded (Bi)simulation on Relational Databases

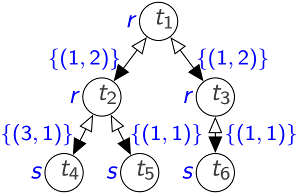
The (bi)simulation relations

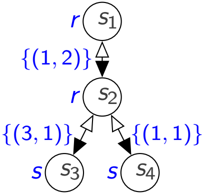
Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

- (lab) it relates only facts with the same relation name, and
- (forth) for every $(r(\overline{a}), r(\overline{b})) \in T$ and every $s(\overline{c}) \in D_1$ there exists $s(\overline{d}) \in D_2$ such that $(s(\overline{c}), s(\overline{d})) \in T$ and $eqtp(\overline{a}, \overline{c}) \subseteq eqtp(\overline{b}, \overline{d})$.

D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$





D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

A guarded simulation of G_2 in G_1:

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

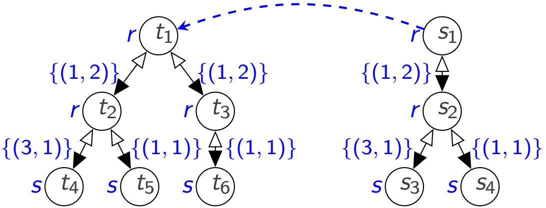
Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

(lab) it relates only facts with the same relation name, and

(forth) for every $(r(\overline{a}), r(\overline{b})) \in T$ and every $s(\overline{c}) \in D_1$ there exists $s(\overline{d}) \in D_2$ such that $(s(\overline{c}), s(\overline{d})) \in T$ and $eqtp(\overline{a}, \overline{c}) \subseteq eqtp(\overline{b}, \overline{d})$.

D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

A guarded simulation of G_2 in G_1:

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

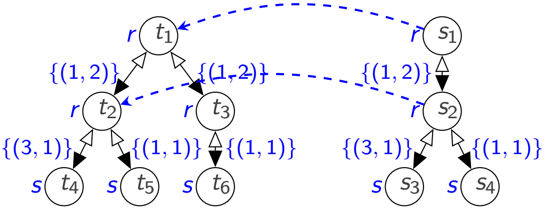
Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

(lab) it relates only facts with the same relation name, and

(forth) for every $(r(\overline{a}), r(\overline{b})) \in T$ and every $s(\overline{c}) \in D_1$ there exists $s(\overline{d}) \in D_2$ such that $(s(\overline{c}), s(\overline{d})) \in T$ and $eqtp(\overline{a}, \overline{c}) \subseteq eqtp(\overline{b}, \overline{d})$.

	D_1
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



	D_2
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

A guarded simulation of G_2 in G_1:

Guarded (Bi)simulation on Relational Databases

The (bi)simulation relations

Definition

A guarded simulation of D_1 in D_2 is a binary relation $T \subseteq D_1 \times D_2$ s.t.

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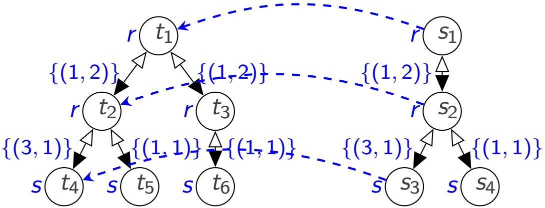
D_1	
t_1	$r(a, b, c)$
t_2	$r(d, a, e)$
t_3	$r(f, a, g)$
t_4	$s(e, h, i)$
t_5	$s(d, j, k)$
t_6	$s(f, l, m)$



D_2	
s_1	$r(n, o, p)$
s_2	$r(q, n, r)$
s_3	$r(r, s, t)$
s_4	$r(q, u, v)$

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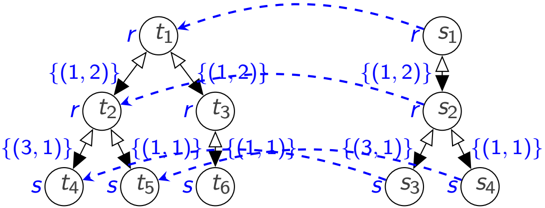

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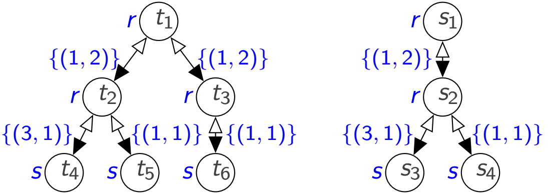

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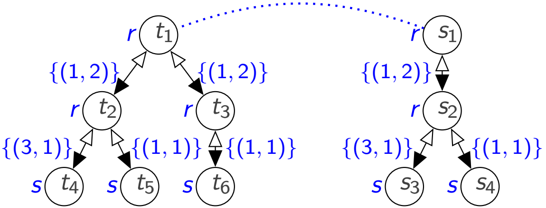

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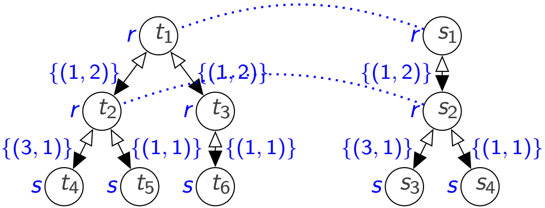

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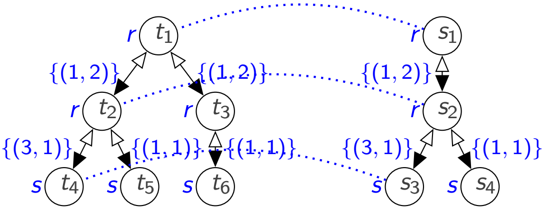

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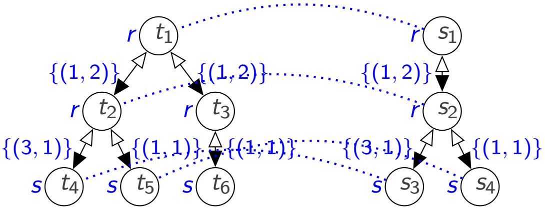

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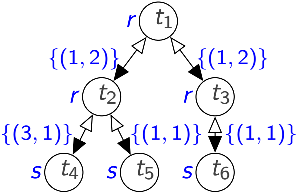

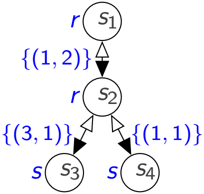

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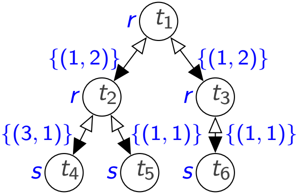

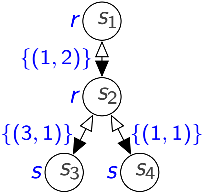

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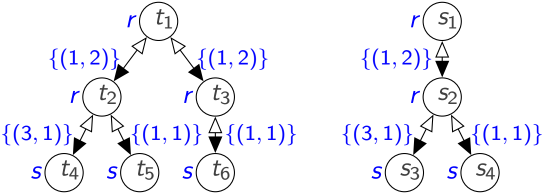

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Inspiring results

For the Guarded Fragment of FOL:

Theorem ([Andréka, Németi & van Benthem 1998][Otto 2012])

The GF is invariant under guarded bisimulation. Moreover, a query expressible in FO is invariant under guarded bisimulation on finite structures if, and only if, it is equivalent in the finite to a query expressible in GF.

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Theorem (Main Result)

FACQs are invariant under guarded simulation. Moreover, a query expressible in FO is invariant under guarded simulation on finite structures if, and only if, it is equivalent in the finite to a union of FACQs.

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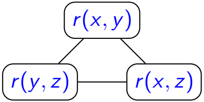

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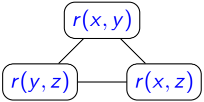

D_1
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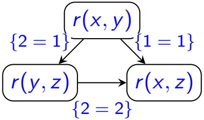

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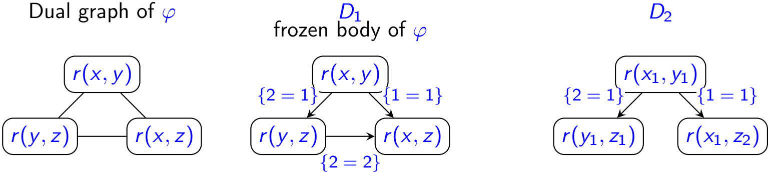

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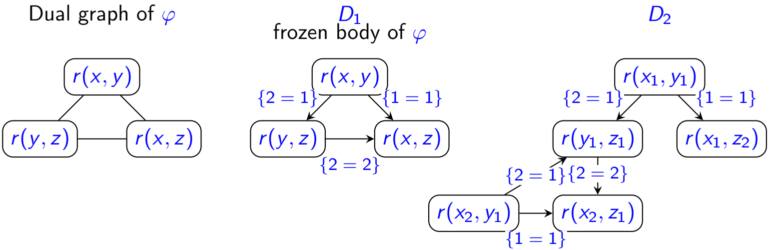

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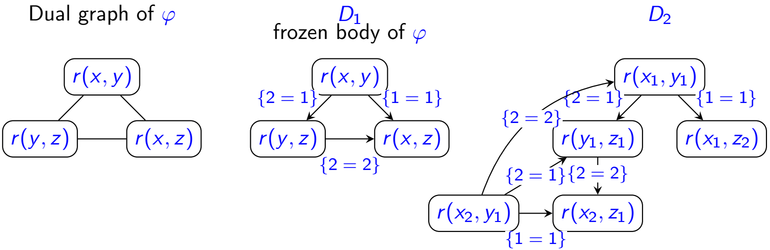

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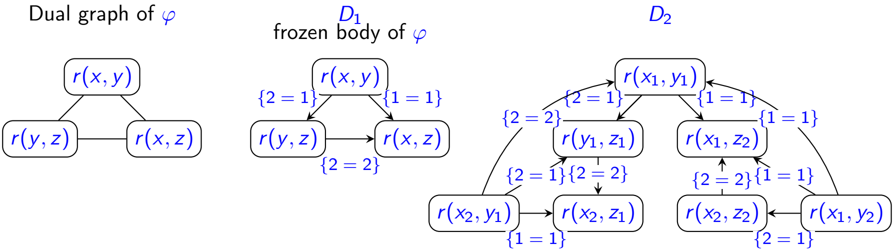

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- This indeed can be shown to be a cover for strict ACQs, i.e., if these are evaluated on $\mathrm{sim}_g(db)$ then from the lab of the retrieved nodes we get the query result up to projection.

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Proposition

Let $k \geq 0$ be a natural number. The following are equivalent.

(1) $db_1, \overline{a} \preceq_f^k db_2, \overline{b}$

(2) *For all FACQs Q of height $\leq k$, if $\overline{a} \in Q(db_1)$ then $\overline{b} \in Q(db_2)$.*

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