

Reasoning about XML Constraints based on XML-to-relational mappings

Matthias Niewerth

Thomas Schwentick

Problem Description

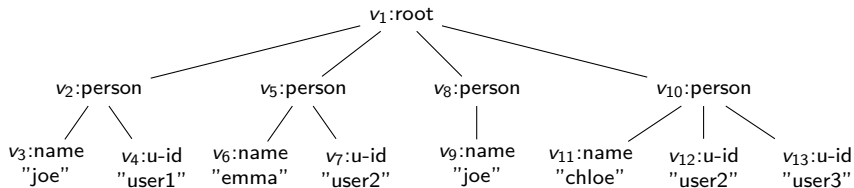


Foundations of XML

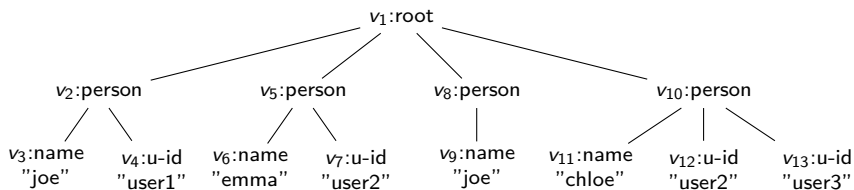
Project Tasks

- Develop a rich modelling language for XML constraints.
- Find an optimal trade-off between structure and data.

Example



Example

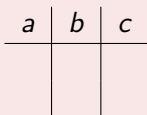


How to express that ...

- ... u-ids are unique?
- ... every person has only one name?
- ... every u-id has at most one associated name?

Contents

Definition



Existing Work

- XFDs
- Hierarchical Constraints
- XML Schema Constraints

Reasoning

- Model-Checking
- Implication Problem

Tree Chase



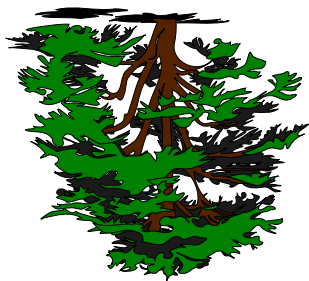
XML-to-relational Constraints



mapping →

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>

XML-to-relational Constraints



mapping \rightarrow

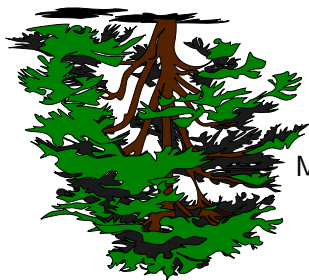
<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>

Definition: XML-to-rel. Constraint

- mapping $m : \mathcal{T} \rightarrow \mathcal{R}$
- relational constraint σ

$$t \models (m, \sigma) \Leftrightarrow m(t) \models \sigma$$

XML-to-relational Constraints



mapping
 $\xrightarrow{\text{MSO, FO, XPath}}$

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Semantical Restriction

Mappings should be invariant under renamings of datavalues.

XML-to-relational Constraints



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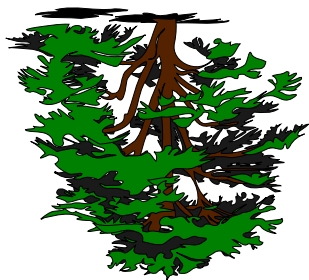
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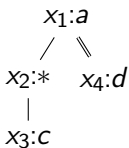
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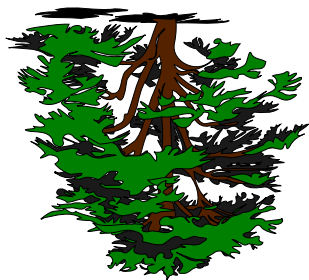
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XML-to-relational Constraints



mapping \rightarrow

$$\begin{array}{c}
 x_1:a \\
 / \quad \parallel \\
 x_2:* \quad x_4:d \\
 | \\
 x_3:c
 \end{array}$$

x_1	$x_1.\circ$	x_2	$x_2.\circ$	x_3	$x_3.\circ$	x_4	$x_4.\circ$

Definition: XML-to-rel. Constraint

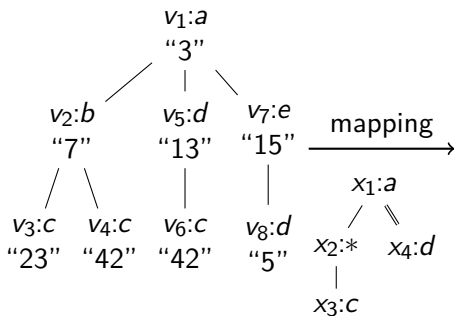
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XML-to-relational Constraints



x_1	$x_{1.\odot}$	x_2	$x_{2.\odot}$	x_3	$x_{3.\odot}$	x_4	$x_{4.\odot}$

Definition: XML-to-rel. Constraint

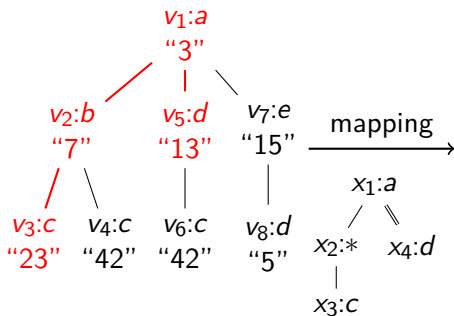
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XML-to-relational Constraints



x_1	$x_{1.\odot}$	x_2	$x_{2.\odot}$	x_3	$x_{3.\odot}$	x_4	$x_{4.\odot}$
v_1	3	v_2	7	v_3	23	v_5	13

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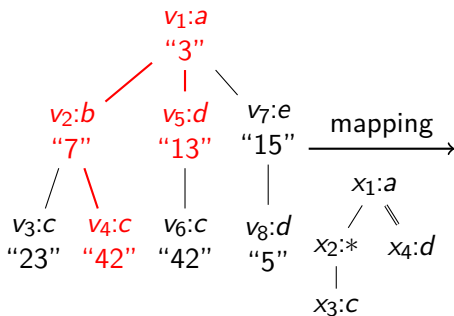
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v_1	3	v_2	7	v_4	42	v_5	13

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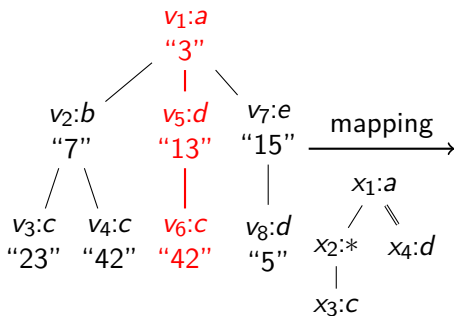
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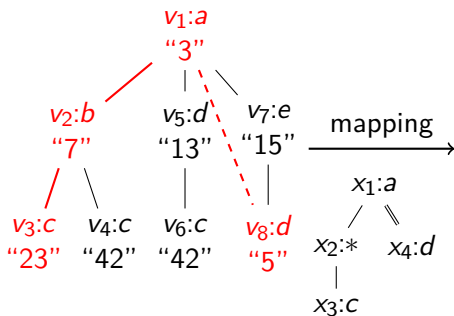
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v_1	3	v_2	7	v_3	23	v_8	5

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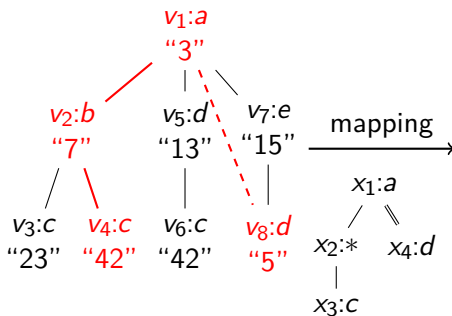
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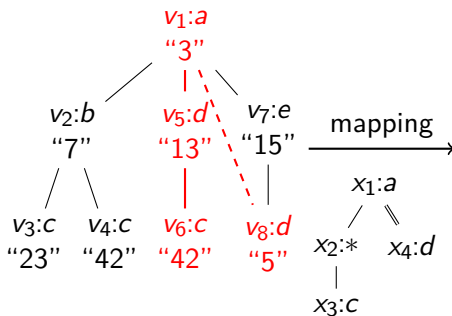
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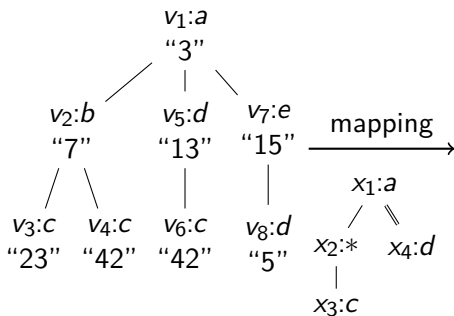
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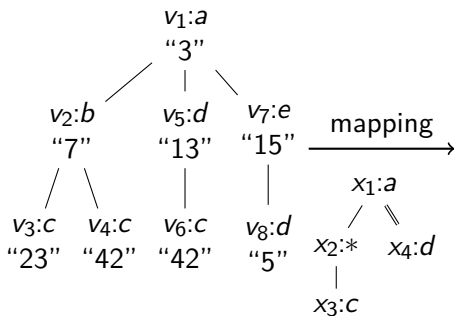
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Functional Dependencies:

XML-to-relational Constraints



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v_1	3	v_2	7	v_3	23	v_5	13
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v_1	3	v_5	13	v_6	42	v_5	13
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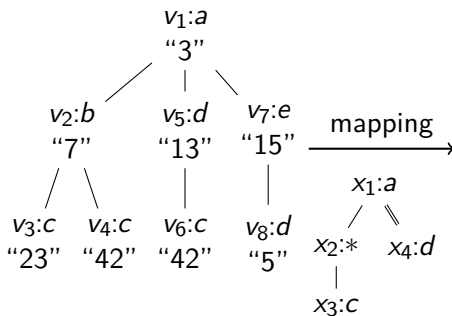
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Functional Dependencies:

- $x_{2.\odot} \rightarrow x_2$

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x_1	$x_{1.\odot}$	x_2	$x_{2.\odot}$	x_3	$x_{3.\odot}$	x_4	$x_{4.\odot}$
v_1	3	v_2	7	v_3	23	v_5	13
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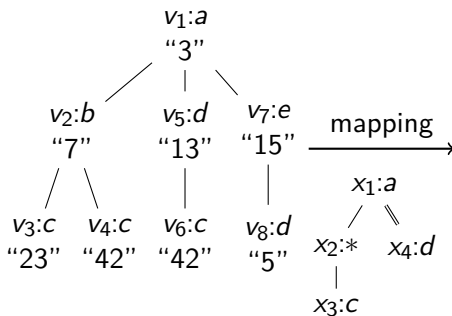
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XML-to-relational Constraints




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v_1	3	v_2	7	v_3	23	v_5	13
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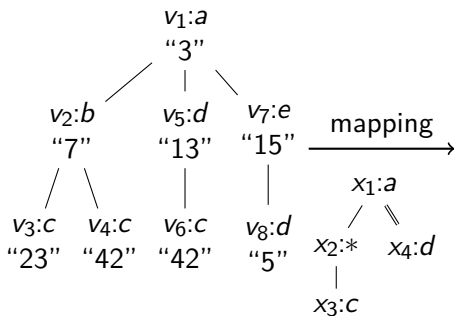
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XML-to-relational Constraints



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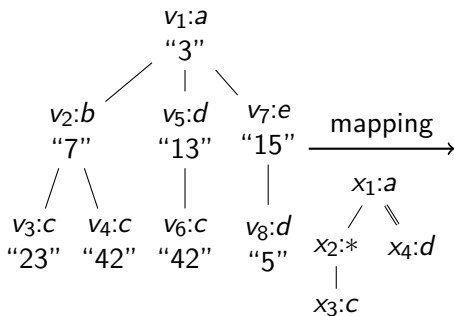
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Functional Dependencies:

- $x_{2.\odot} \rightarrow x_2$
- $x_{3.\odot} \rightarrow x_3$

XML-to-relational Constraints



x_1	$x_{1.\text{@}}$	x_2	$x_{2.\text{@}}$	x_3	$x_{3.\text{@}}$	x_4	$x_{4.\text{@}}$
v_1	3	v_2	7	v_3	23	v_5	13
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v_1	3	v_5	13	v_6	42	v_5	13
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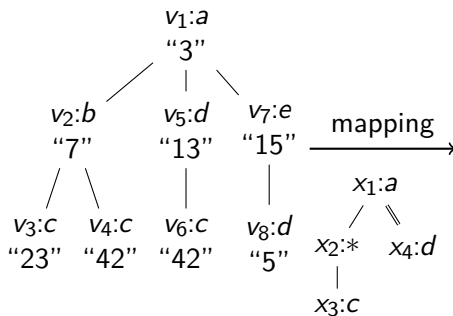
- mapping $m : \mathcal{T} \rightarrow \mathcal{R}$
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$$t \models (m, \sigma) \Leftrightarrow m(t) \models \sigma$$

Functional Dependencies:

- $x_{2.\text{@}} \rightarrow x_2$
- $x_{3.\text{@}} \rightarrow x_3$
- $x_2, x_{3.\text{@}} \rightarrow x_3$

XML-to-relational Constraints



x_1	$x_{1.\text{@}}$	x_2	$x_{2.\text{@}}$	x_3	$x_{3.\text{@}}$	x_4	$x_{4.\text{@}}$
v_1	3	v_2	7	v_3	23	v_5	13
v_1	3	v_2	7	v_4	42	v_5	13
v_1	3	v_5	13	v_6	42	v_5	13
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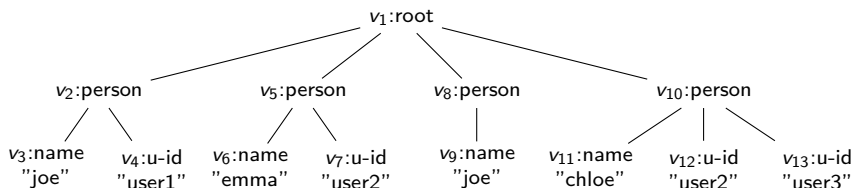
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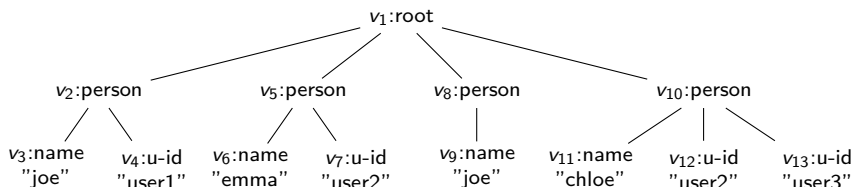
Functional Dependencies:

- $x_{2.\text{@}} \rightarrow x_2$
- $x_{3.\text{@}} \rightarrow x_3$
- $x_2, x_{3.\text{@}} \rightarrow x_3$

Example



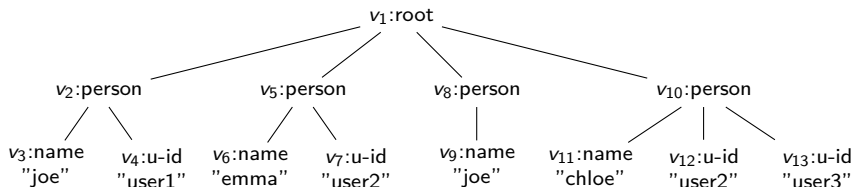
Example



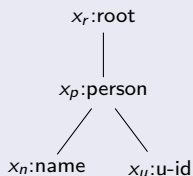
How to express that ...

- ... u-ids are unique?
- ... every person has only one name (node)?
- ... every u-id has at most one associated name (data value)?

Example



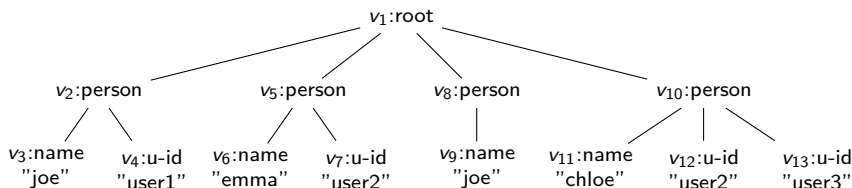
Pattern/Mapping



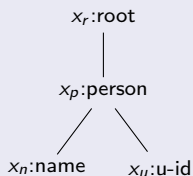
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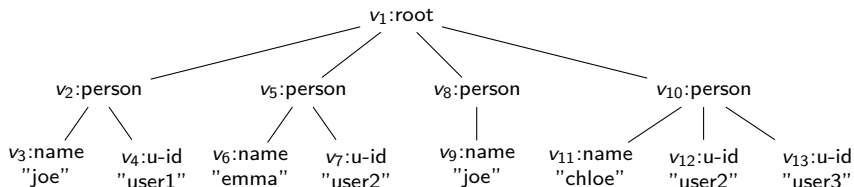
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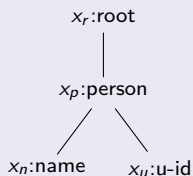
How to express that ...

- ... u-ids are unique?
 $X_U.\text{@} \rightarrow X_U$
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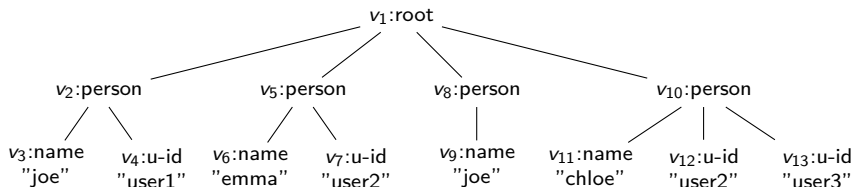
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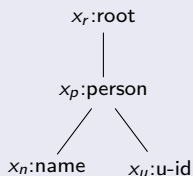
How to express that ...

- ... u-ids are unique?
 $X_U.\@ \rightarrow X_U$
- ... every person has only one name (node)?
 $X_P \rightarrow X_N$
- ... every u-id has at most one associated name (data value)?

Example



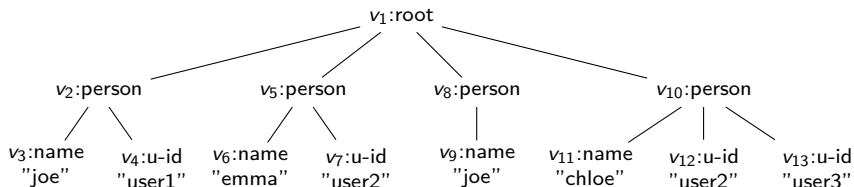
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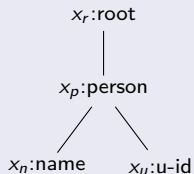
How to express that ...

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 $X_U.\text{@} \rightarrow X_U$
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 $X_P \rightarrow X_N$
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 $X_U.\text{@} \rightarrow X_N.\text{@}$

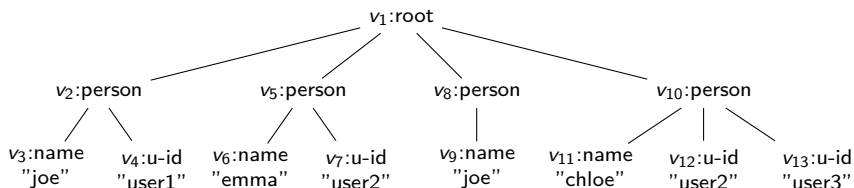
Incomplete/Missing Data



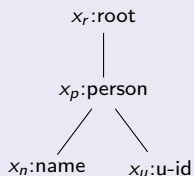
Pattern/Mapping



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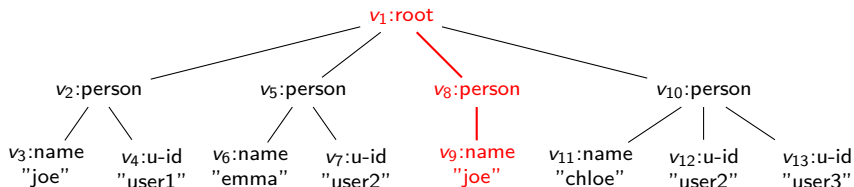
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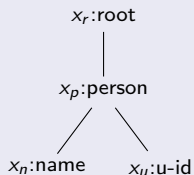
Relation

x_r	x_p	x_n	$x_n \cdot @$	x_u	$x_u \cdot @$
v_1	v_2	v_3	joe	v_4	user1
v_1	v_5	v_6	emma	v_7	user2
v_1	v_{10}	v_{11}	chloe	v_{12}	user2
v_1	v_{10}	v_{11}	chloe	v_{13}	user3

Incomplete/Missing Data



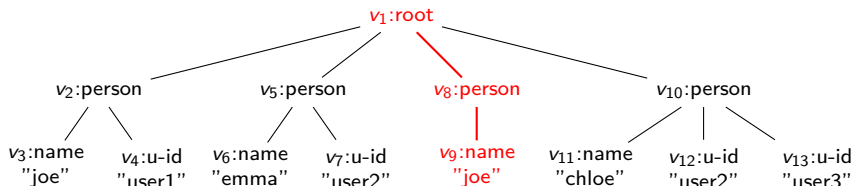
Pattern/Mapping



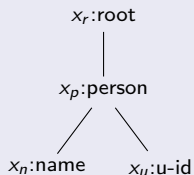
Relation

x_r	x_p	x_n	$x_n \cdot @$	x_u	$x_u \cdot @$
v_1	v_2	v_3	joe	v_4	user1
v_1	v_5	v_6	emma	v_7	user2
v_1	v_{10}	v_{11}	chloe	v_{12}	user2
v_1	v_{10}	v_{11}	chloe	v_{13}	user3

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Pattern/Mapping



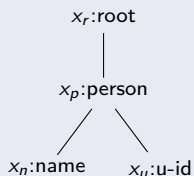
Relation

x_r	x_p	x_n	$x_n \cdot @$	x_u	$x_u \cdot @$
v_1	v_2	v_3	joe	v_4	user1
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Incomplete/Missing Data

Semantics

Pattern/Mapping



Relation

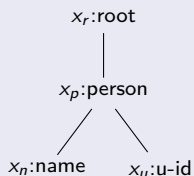
x_r	x_p	x_n	$x_n.\textcircled{\circ}$	x_u	$x_u.\textcircled{\circ}$
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v_1	v_5	v_6	emma	v_7	user2
v_1	v_8	v_9	joe	\perp	\perp
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Semantics

- "Complete Data Semantics":
One tuple for every **complete** embedding of the tree pattern

Pattern/Mapping



Relation

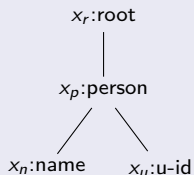
x_r	x_p	x_n	$x_n \cdot @$	x_u	$x_u \cdot @$
v_1	v_2	v_3	joe	v_4	user1
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Incomplete/Missing Data

Semantics

- "Complete Data Semantics":
One tuple for every **complete** embedding of the tree pattern
- "Incomplete Data Semantics":
One tuple for every **maximal** embedding of the tree pattern

Pattern/Mapping



Relation

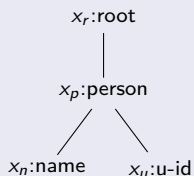
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Incomplete/Missing Data

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Pattern/Mapping



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In this Paper

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- "Incomplete Data Semantics" to compare with other frameworks.

Incomplete/Missing Data

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Incomplete/Missing Data

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In this Paper

- "Incomplete Data Semantics" to compare with other frameworks.
- "Complete Data Semantics" for complexity results.
- Full version of paper will use "Incomplete Data Semantics"

Functional Dependencies vs. Key Constraints

Functional Dependencies vs. Key Constraints

Informal Definition

A key constraint uniquely identifies a real-world entity.

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In XML documents real-world entities correspond to nodes.

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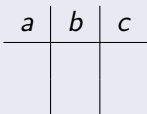
In XML documents real-world entities correspond to nodes.

Formal Definition: XKFD

An XML Key Functional Dependency (XKFD) is an FD $(m, A \rightarrow b)$, where b refers to a node.

Contents

Definition



Existing Work

- XFDs
- Hierarchical Constraints
- XML Schema Constraints

Reasoning

- Model-Checking
- Implication Problem

Tree Chase



XML Functional Dependencies (XFDs)

- Introduced by Arenas, Fan and Libkin 2002

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Syntax/Example

Set of PATHs	→	PATH
{/root/person/u-id.®}	→	/root/person/u-id

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Semantics (in our framework)

- Combine all paths into a tree pattern.
- Use the canonical FD

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Set of PATHs → PATH
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Definition [Arenas, Fan, Libkin 2002]

An XML Database is in XML Normal Form, if all FDs are XKFDs.

Hierarchical Key Constraints

Hierarchical Key Constraints

- Introduced by Buneman et al. 2001

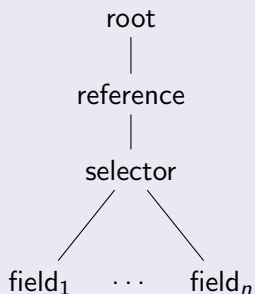
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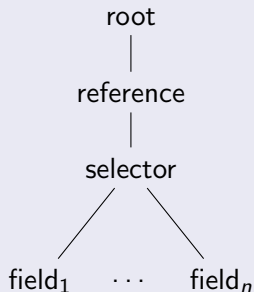


$\text{reference}, \text{field}_1.\text{@}, \dots, \text{field}_n.\text{@} \rightarrow \text{selector}$

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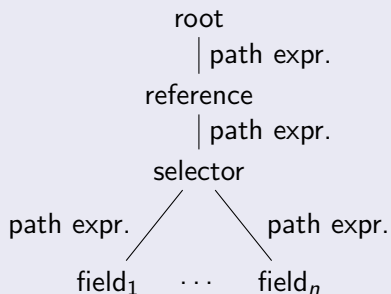
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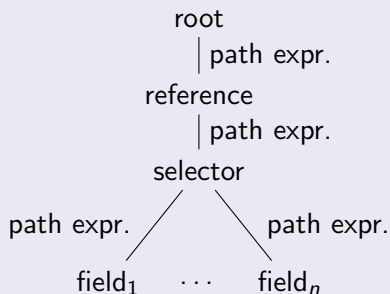
reference, field₁.@, ..., field_n.@ → selector

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- Path Expressions instead of simple paths

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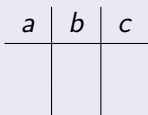


reference, field₁.@, ..., field_n.@ → selector

- All hierarchical key constraints are XKFDs.
- Path Expressions instead of simple paths
- Can be expressed with tree patterns if Linear XPath is used.

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Algorithmic Problems

Model Checking Problem: $MC(\mathcal{M}, \mathcal{C})$

Given: XML tree t ,

XML-to-rel constraint $(m, \sigma) \in \mathcal{M} \times \mathcal{C}$

Question: Does $t \models (m, \sigma)$ hold?

Algorithmic Problems

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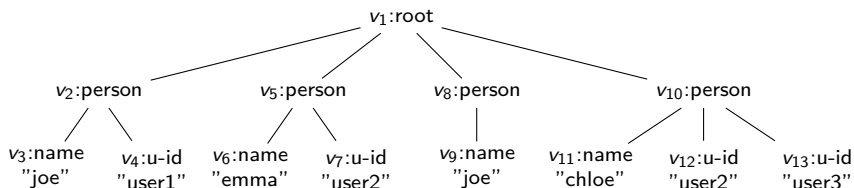
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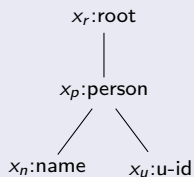
Theorem

$MC(TP[//, //, *], FD)$ can be solved in PTIME

Model Checking



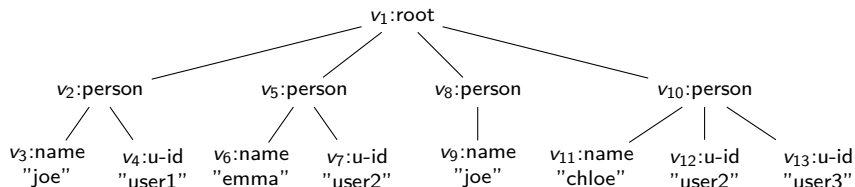
Pattern/Mapping



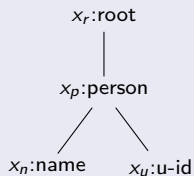
Functional Dependency

- U-IDs are unique: $x_u.@ \rightarrow x_u$

Model Checking



Pattern/Mapping



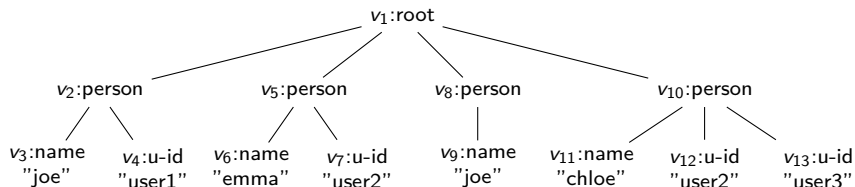
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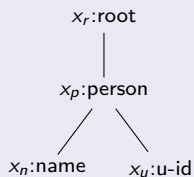
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x_r	x_p	x_n	$x_{n.\text{@}}$	x_u	$x_{u.\text{@}}$
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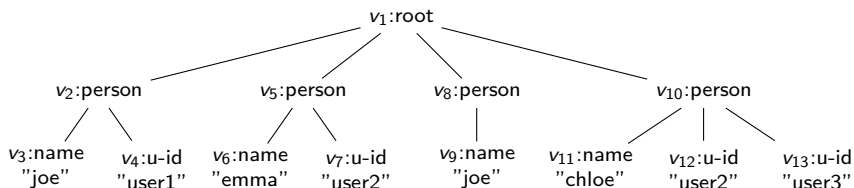
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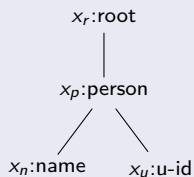
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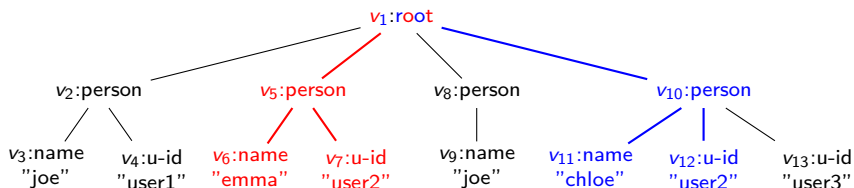
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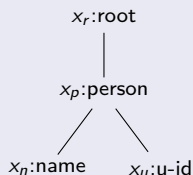
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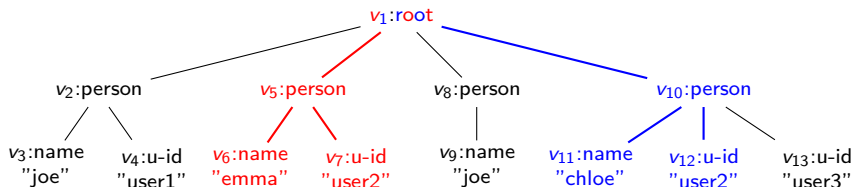
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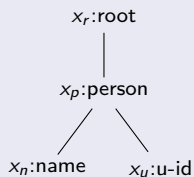
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Model Checking



Pattern/Mapping



Functional Dependency

- U-IDs are unique: $x_{u.\text{@}} \rightarrow x_u$
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Complexity

Combined Complexity: $\in \text{DTIME}(|t|^2|p|)$
 Data Complexity: LOGSPACE

The Implication Problem

Problem: $\text{IMP}(\mathcal{M}, \mathcal{C}, \mathcal{S})$

Given: XML-to-rel. constraints

$(m_1, \sigma_1), \dots, (m_n, \sigma_n)$ and $(m, \sigma) \in \mathcal{M} \times \mathcal{C}$,
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$\text{IMP}(\text{TP}[/, //, *], \text{FD}, \text{finite alphabet})$ is *undecidable*.

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$\text{IMP}(\text{TP}[/, //, *], \text{FD}, \text{finite alphabet})$ is *undecidable*.

Theorem

$\text{IMP}(\text{MSO}, \text{XKFD}, \text{MSO})$ is *decidable*.

MSO formulas have no access to data values!

The Implication Problem

Theorem

IMP(MSO, XKFD, MSO) is decidable.

The Implication Problem

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IMP(MSO, XKFD, MSO) is decidable.

Proof

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- A counterexample to an XKFD needs at most linearly many **important** data values, which occur **more than once**.

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- A counterexample to an XKFD needs at most linearly many **important** data values, which occur **more than once**.
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Proof

- A counterexample to an XKFD needs at most linearly many **important** data values, which occur **more than once**.
- Encode **important** data values in the labels.
- MSO (without data values) is decidable on trees.

Results

Results

	TP[//]	TP[//,*]	TP[//, //]	TP[//, //,*]
FD	$\in P$	$\in P$	co-NP	co-NP-hard

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FD + Schema	$\in P$	co-NP-hard $\in EXPTIME$	co-NP-hard	undecidable

Schema means:

- finite alphabet for lower bounds
- simple DTDs for upper bounds

Results

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	TP[//]	TP[//, *]	TP[//, //]	TP[//, //, *]
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Upper Bounds:

- chase algorithm
- small (encodings of) counter examples

Results

Results

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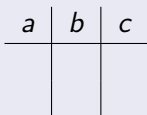
- chase algorithm
- small (encodings of) counter examples

Lower Bounds:

- reductions

Contents

Definition



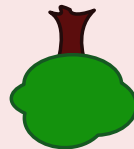
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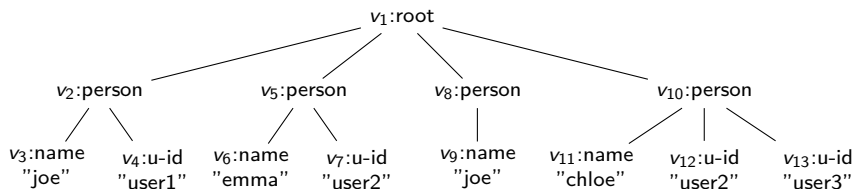
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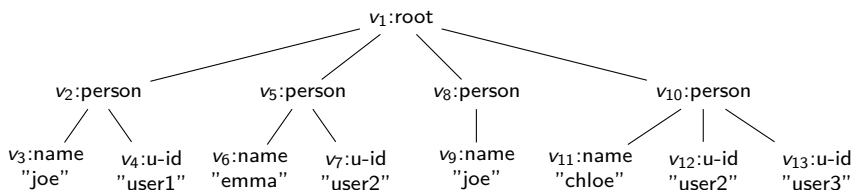
Tree Chase



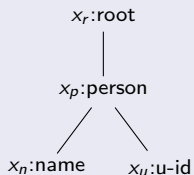
Our Tree Chase



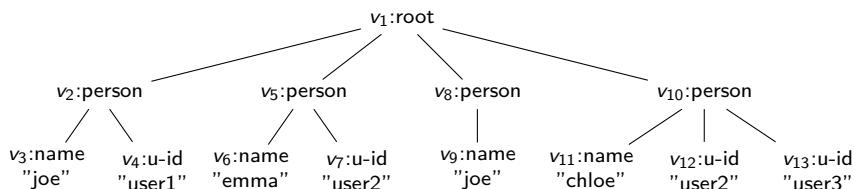
Our Tree Chase



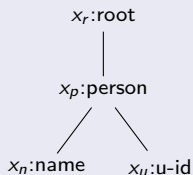
Pattern/Mapping



Our Tree Chase



Pattern/Mapping



Implication Instance

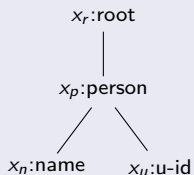
$$\Sigma = \{x_u.\text{@} \rightarrow x_u, x_p \rightarrow x_n\}$$

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$$\Sigma \models \sigma?$$

Our Tree Chase

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Implication Instance

$$\Sigma = \{x_u.\text{@} \rightarrow x_u, x_p \rightarrow x_n\}$$

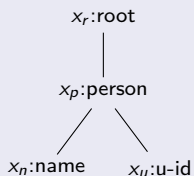
$$\sigma = x_u.\text{@} \rightarrow x_n.\text{@}$$

$$\Sigma \models \sigma?$$

Our Tree Chase

Relation

Pattern/Mapping



Implication Instance

$$\Sigma = \{x_u.\textcircled{\ast} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.\textcircled{\ast} \rightarrow x_n.\textcircled{\ast}$$

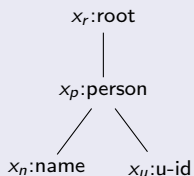
$$\Sigma \models \sigma?$$

Our Tree Chase

Relation

x_r	x_p	x_n	$x_n.\textcircled{a}$	x_u	$x_u.\textcircled{a}$

Pattern/Mapping



Implication Instance

$$\Sigma = \{x_u.\textcircled{a} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.\textcircled{a} \rightarrow x_n.\textcircled{a}$$

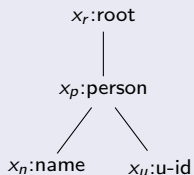
$$\Sigma \models \sigma?$$

Our Tree Chase

Relation

x_r	x_p	x_n	$x_{n.\textcircled{e}}$	x_u	$x_{u.\textcircled{e}}$
v_1	v_2	v_3	1	v_4	2
v_5	v_6	v_7	3	v_8	2

Pattern/Mapping



Implication Instance

$$\Sigma = \{x_{u.\textcircled{e}} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_{u.\textcircled{e}} \rightarrow x_{n.\textcircled{e}}$$

$$\Sigma \models \sigma?$$

Our Tree Chase

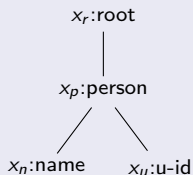
Tree



Relation

x_r	x_p	x_n	$x_{n.\textcircled{e}}$	x_u	$x_{u.\textcircled{e}}$
v_1	v_2	v_3	1	v_4	2
v_5	v_6	v_7	3	v_8	2

Pattern/Mapping



Implication Instance

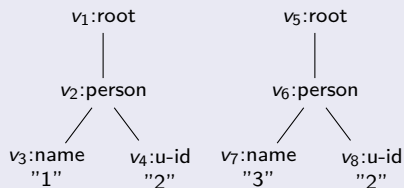
$$\Sigma = \{x_{u.\textcircled{e}} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_{u.\textcircled{e}} \rightarrow x_{n.\textcircled{e}}$$

$$\Sigma \models \sigma?$$

Our Tree Chase

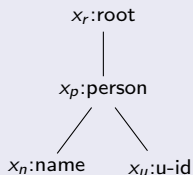
Tree



Relation

x_r	x_p	x_n	$x_n.\textcircled{\bullet}$	x_u	$x_u.\textcircled{\bullet}$
v_1	v_2	v_3	1	v_4	2
v_5	v_6	v_7	3	v_8	2

Pattern/Mapping



Implication Instance

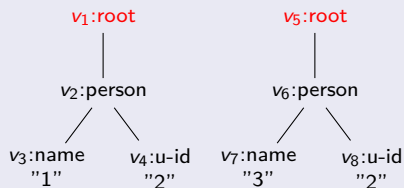
$$\Sigma = \{x_u.\textcircled{\bullet} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.\textcircled{\bullet} \rightarrow x_n.\textcircled{\bullet}$$

$$\Sigma \models \sigma?$$

Our Tree Chase

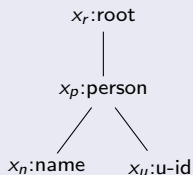
Tree



Relation

x_r	x_p	x_n	$x_n.\textcircled{\cdot}$	x_u	$x_u.\textcircled{\cdot}$
v1	v2	v3	1	v4	2
v5	v6	v7	3	v8	2

Pattern/Mapping



Implication Instance

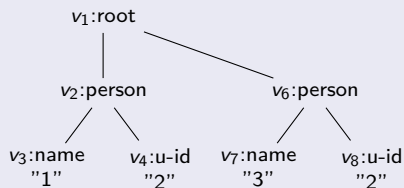
$$\Sigma = \{x_u.\textcircled{\cdot} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.\textcircled{\cdot} \rightarrow x_n.\textcircled{\cdot}$$

$$\Sigma \models \sigma?$$

Our Tree Chase

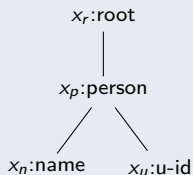
Tree



Relation

x_r	x_p	x_n	$x_n.\textcircled{\ast}$	x_u	$x_u.\textcircled{\ast}$
v_1	v_2	v_3	1	v_4	2
v_1	v_6	v_7	3	v_8	2

Pattern/Mapping



Implication Instance

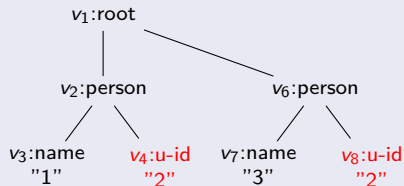
$$\Sigma = \{x_u.\textcircled{\ast} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.\textcircled{\ast} \rightarrow x_n.\textcircled{\ast}$$

$$\Sigma \models \sigma?$$

Our Tree Chase

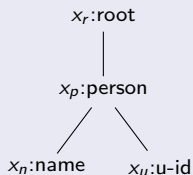
Tree



Relation

x_r	x_p	x_n	$x_{n.\textcircled{}}$	x_u	$x_{u.\textcircled{}}$
v_1	v_2	v_3	1	v_4	2
v_1	v_6	v_7	3	v_8	2

Pattern/Mapping



Implication Instance

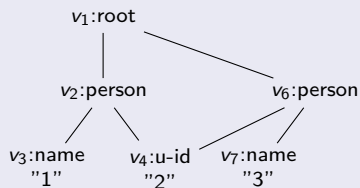
$$\Sigma = \{x_{u.\textcircled{}} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_{u.\textcircled{}} \rightarrow x_{n.\textcircled{}}$$

$$\Sigma \models \sigma?$$

Our Tree Chase

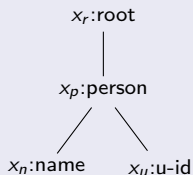
Tree



Relation

x_r	x_p	x_n	$x_n.@$	x_u	$x_u.@$
v_1	v_2	v_3	1	v_4	2
v_1	v_6	v_7	3	v_4	2

Pattern/Mapping



Implication Instance

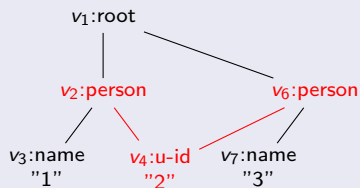
$$\Sigma = \{x_u.@ \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.@ \rightarrow x_n.@$$

$$\Sigma \models \sigma?$$

Our Tree Chase

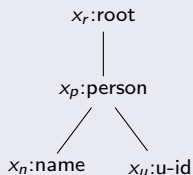
Tree



Relation

x_r	x_p	x_n	$x_n.@$	x_u	$x_u.@$
v_1	v_2	v_3	1	v_4	2
v_1	v_6	v_7	3	v_4	2

Pattern/Mapping



Implication Instance

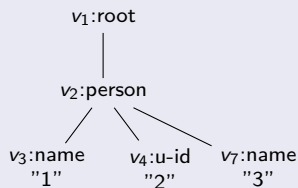
$$\Sigma = \{x_u.@ \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.@ \rightarrow x_n.@$$

$$\Sigma \models \sigma?$$

Our Tree Chase

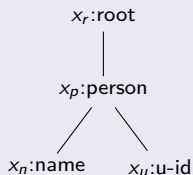
Tree



Relation

x_r	x_p	x_n	$x_n.\textcircled{\ast}$	x_u	$x_u.\textcircled{\ast}$
v_1	v_2	v_3	1	v_4	2
v_1	v_2	v_7	3	v_4	2

Pattern/Mapping



Implication Instance

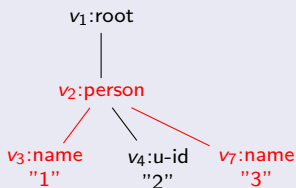
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$$\sigma = x_u.\textcircled{\ast} \rightarrow x_n.\textcircled{\ast}$$

$$\Sigma \models \sigma?$$

Our Tree Chase

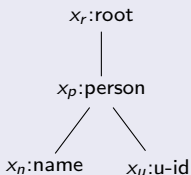
Tree



Relation

x_r	x_p	x_n	$x_n.\textcircled{\ast}$	x_u	$x_u.\textcircled{\ast}$
v_1	v_2	v_3	1	v_4	2
v_1	v_2	v_7	3	v_4	2

Pattern/Mapping



Implication Instance

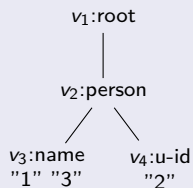
$$\Sigma = \{x_u.\textcircled{\ast} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.\textcircled{\ast} \rightarrow x_n.\textcircled{\ast}$$

$$\Sigma \models \sigma?$$

Our Tree Chase

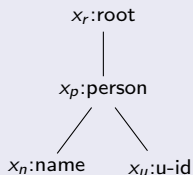
Tree



Relation

x_r	x_p	x_n	$x_n.@$	x_u	$x_u.@$
v_1	v_2	v_3	1	v_4	2
v_1	v_2	v_3	3	v_4	2

Pattern/Mapping



Implication Instance

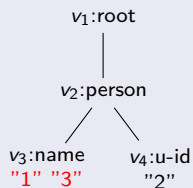
$$\Sigma = \{x_u.@ \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.@ \rightarrow x_n.@$$

$$\Sigma \models \sigma?$$

Our Tree Chase

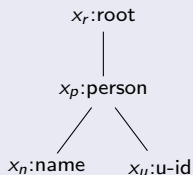
Tree



Relation

x_r	x_p	x_n	$x_n.@$	x_u	$x_u.@$
v_1	v_2	v_3	1	v_4	2
v_1	v_2	v_3	3	v_4	2

Pattern/Mapping



Implication Instance

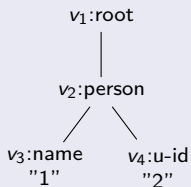
$$\Sigma = \{x_u.@ \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.@ \rightarrow x_n.@$$

$$\Sigma \models \sigma?$$

Our Tree Chase

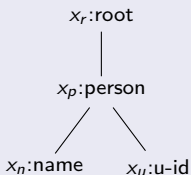
Tree



Relation

x_r	x_p	x_n	$x_n.@$	x_u	$x_u.@$
v_1	v_2	v_3	1	v_4	2
v_1	v_2	v_3	1	v_4	2

Pattern/Mapping



Implication Instance

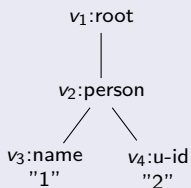
$$\Sigma = \{x_u.@ \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.@ \rightarrow x_n.@$$

$$\Sigma \models \sigma?$$

Our Tree Chase

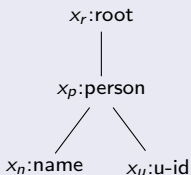
Tree



Relation

x_r	x_p	x_n	$x_{n.\textcircled{}}$	x_u	$x_{u.\textcircled{}}$
v_1	v_2	v_3	1	v_4	2

Pattern/Mapping



Implication Instance

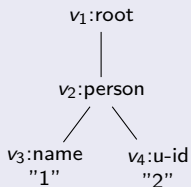
$$\Sigma = \{x_{u.\textcircled{}} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_{u.\textcircled{}} \rightarrow x_{n.\textcircled{}}$$

$$\Sigma \models \sigma?$$

Our Tree Chase

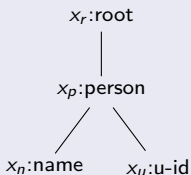
Tree



Relation

x_r	x_p	x_n	$x_n.\textcircled{e}$	x_u	$x_u.\textcircled{e}$
v_1	v_2	v_3	1	v_4	2

Pattern/Mapping



Implication Instance

$$\Sigma = \{x_u.\textcircled{e} \rightarrow x_u, x_p \rightarrow x_n\}$$

$$\sigma = x_u.\textcircled{e} \rightarrow x_n.\textcircled{e}$$

$$\Sigma \models \sigma? \quad \checkmark$$

Results & Further Work

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- Defined a general framework for XML Constraints

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- Showed how existing work integrates into the framework

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Complexity Results for Implication

	TP[//]	TP[/, *]	TP[/, //]	TP[/, //, *]
XKFD		$\in P$	co-NP	co-NP
FD		$\in P$	co-NP	co-NP-hard
XKFD + Schema	$\in P$	co-NP	co-NP	PSPACE
FD + Schema		co-NP-hard $\in EXPTIME$	co-NP-hard	undecidable

Results & Further Work

- Defined a general framework for XML Constraints
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Complexity Results for Implication

	TP[//]	TP[/, *]	TP[/, //]	TP[/, //, *]
XKFD	∈ P	∈ P	co-NP	co-NP
FD		∈ P	co-NP	co-NP-hard
XKFD + Schema		co-NP	co-NP	PSPACE
FD + Schema		co-NP-hard ∈ EXPTIME	co-NP-hard	undecidable

Further Work

- Solve missing complexities
- Incomplete Data
- More general schemas
- Connections to XML Data Exchange