Dynamic Conjunctive Queries

Thomas Schwentick und Thomas Zeume

ICDT

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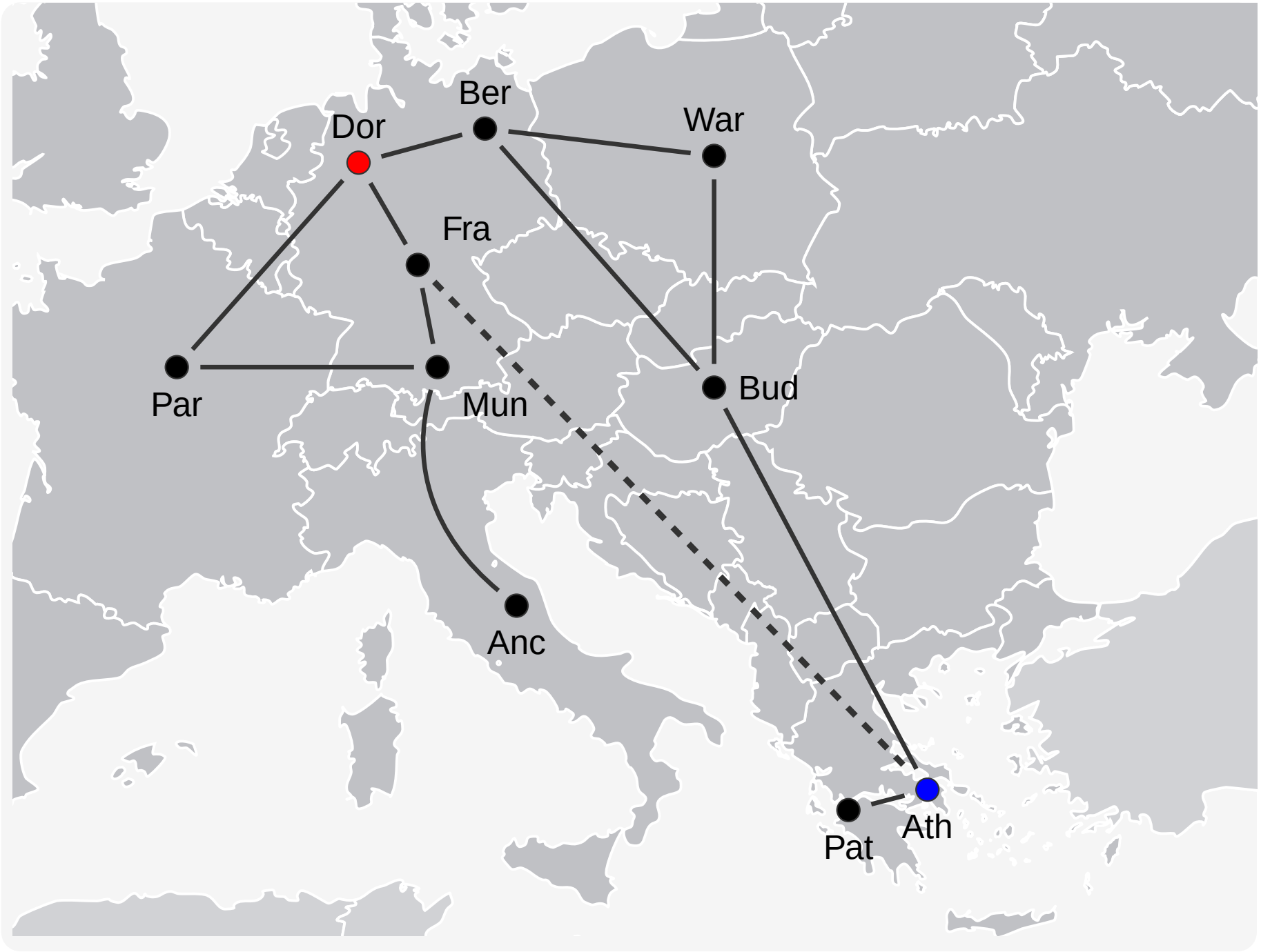
technische universität dortmund

Lehrstuhl Logik in der Informatik

How to go to Athens...



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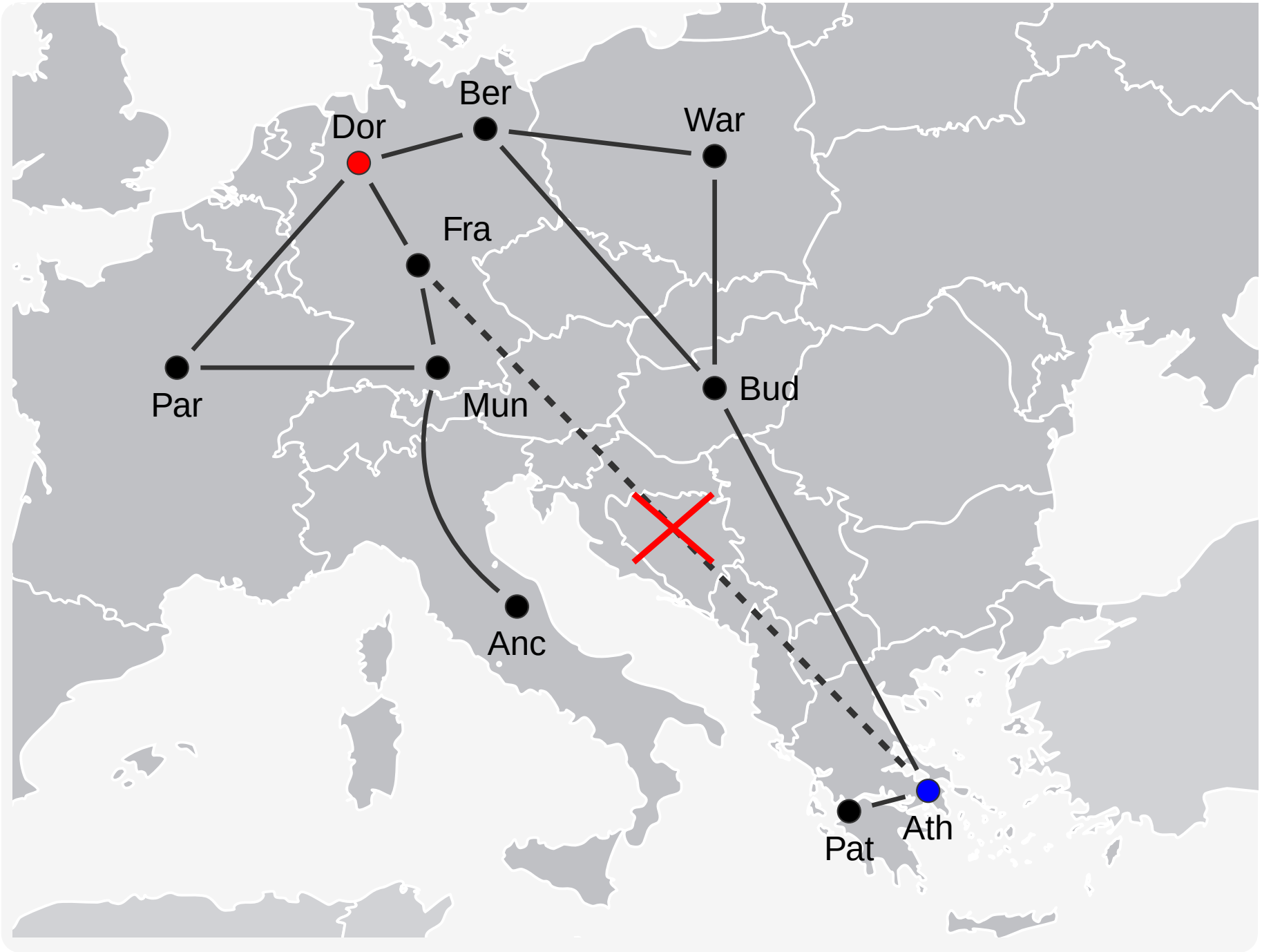

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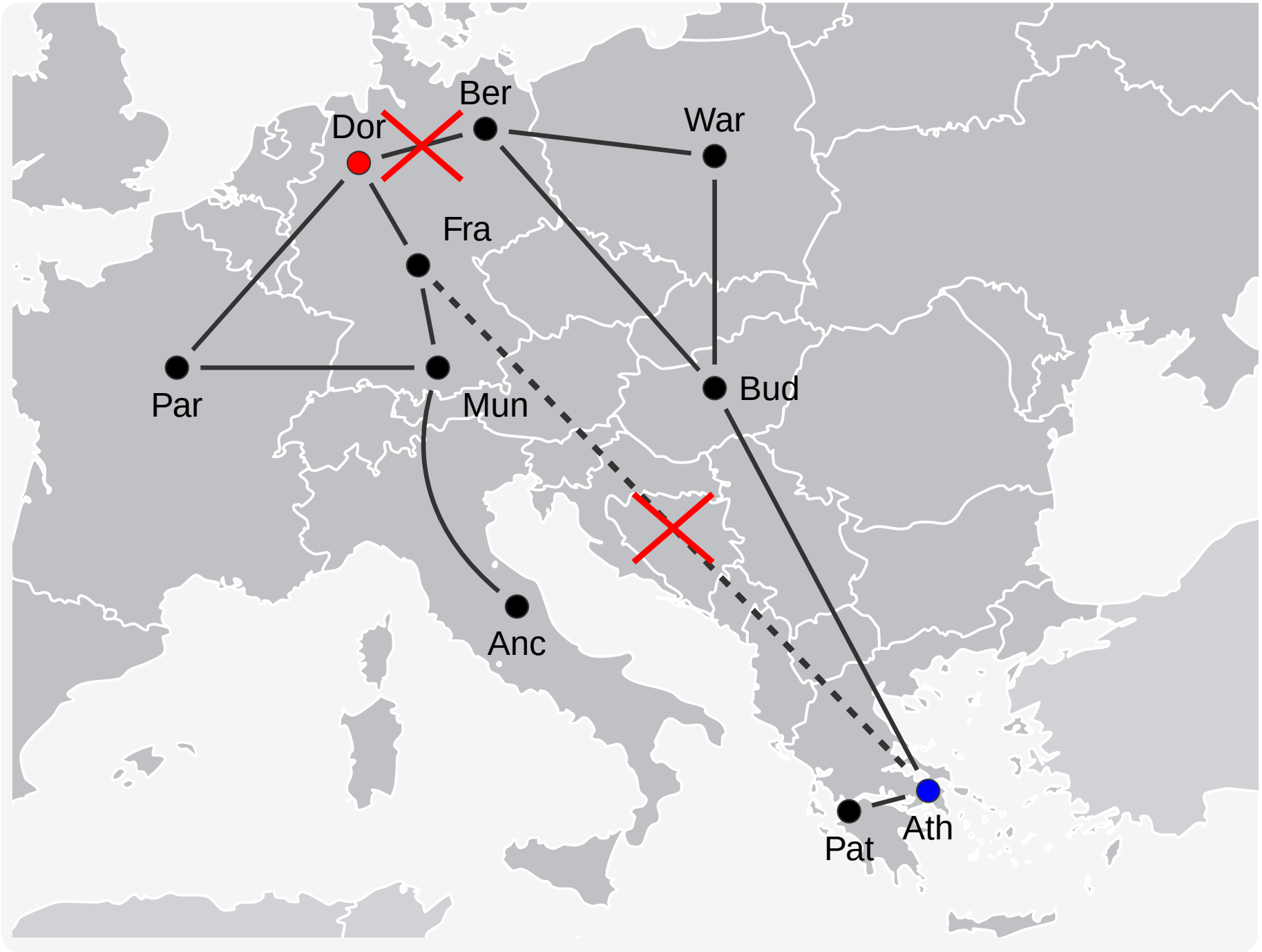

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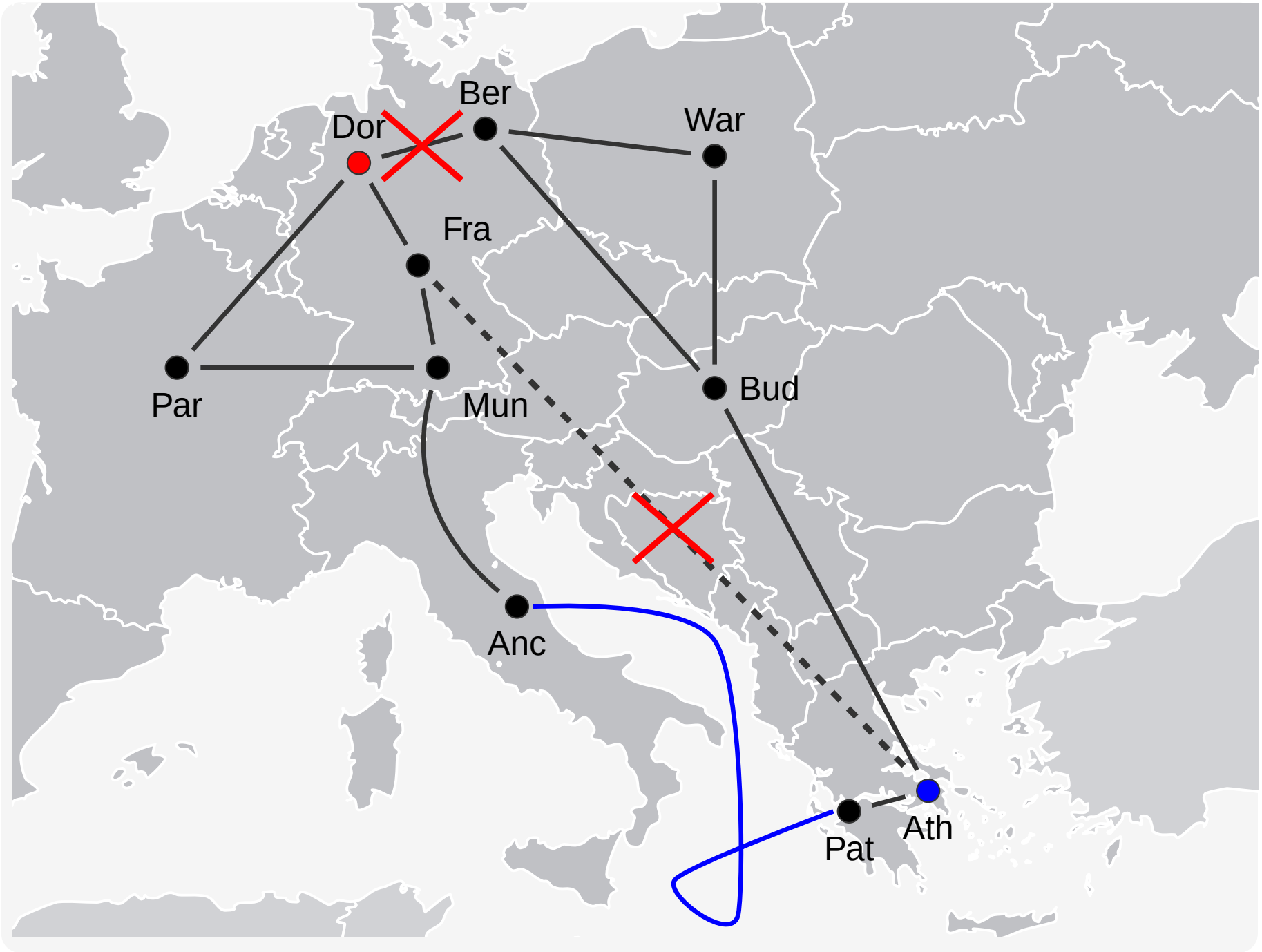

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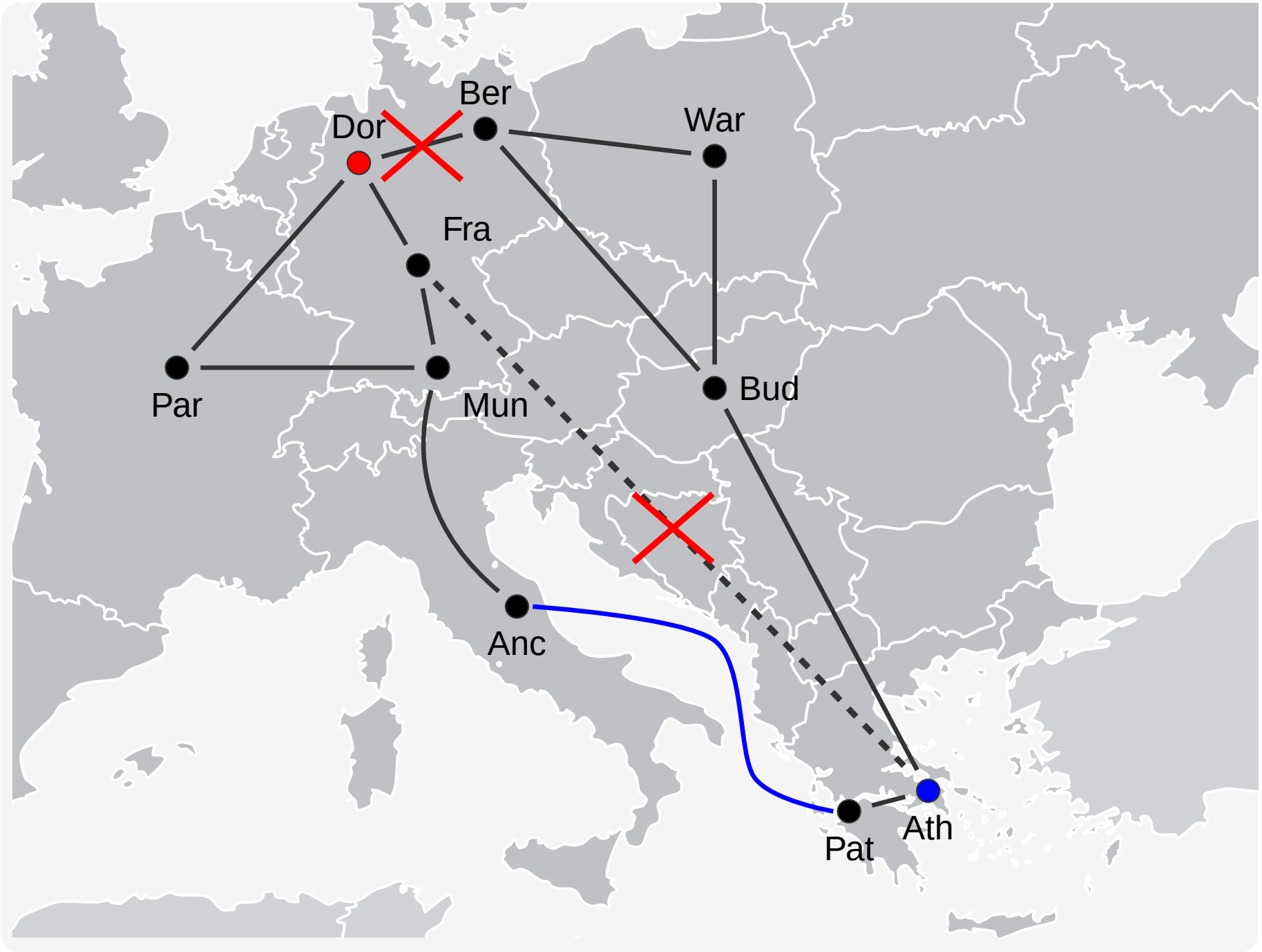
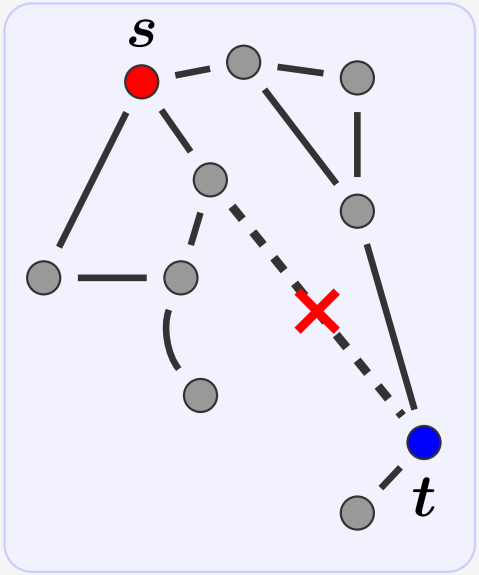


The Dynamic Approach



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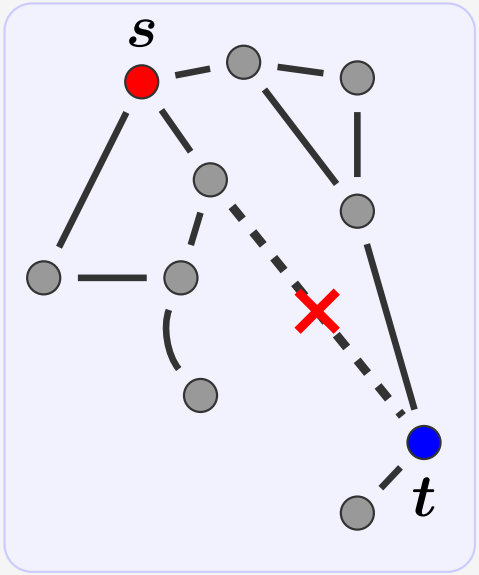

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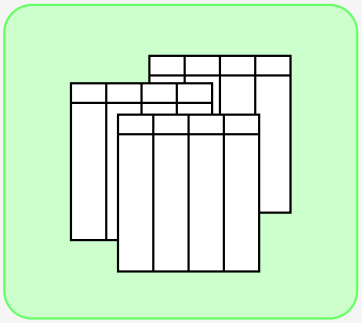

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Reach(s,t)?

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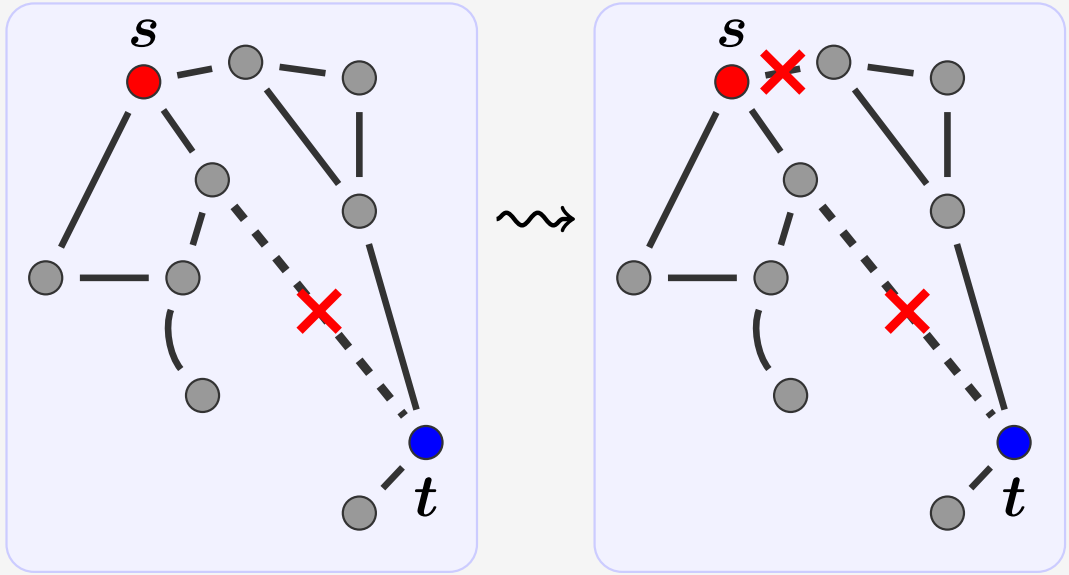

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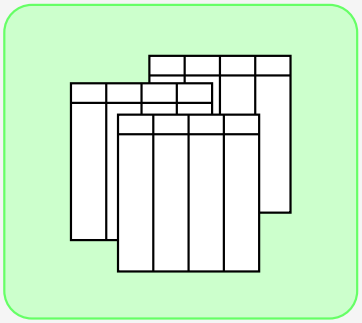

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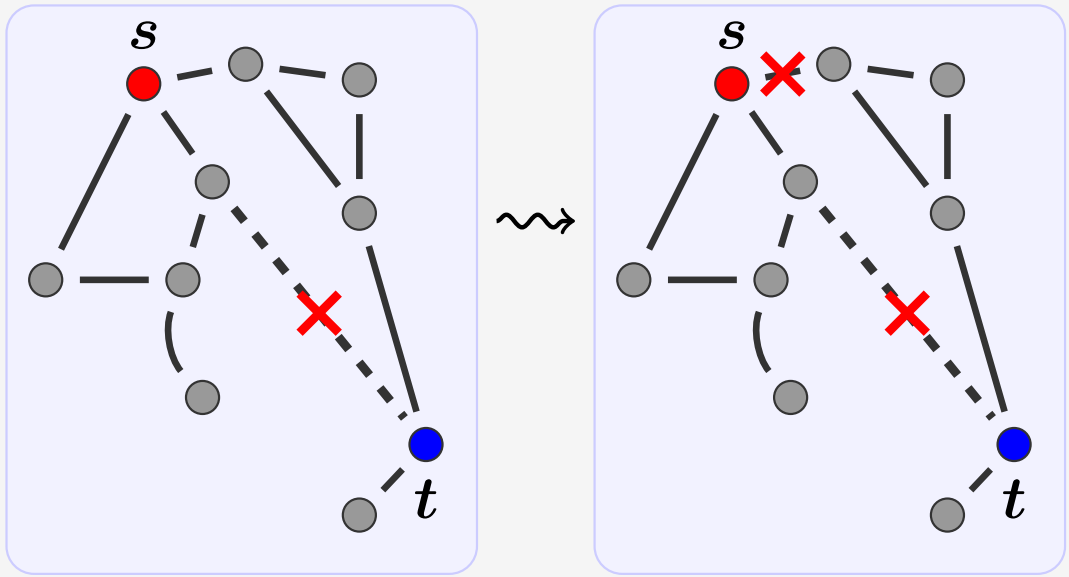

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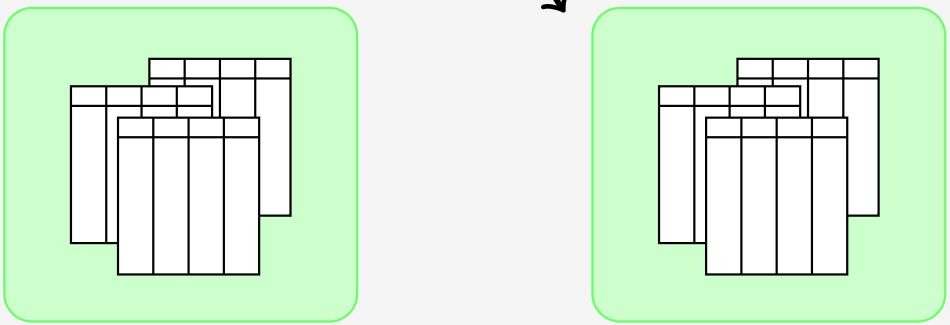

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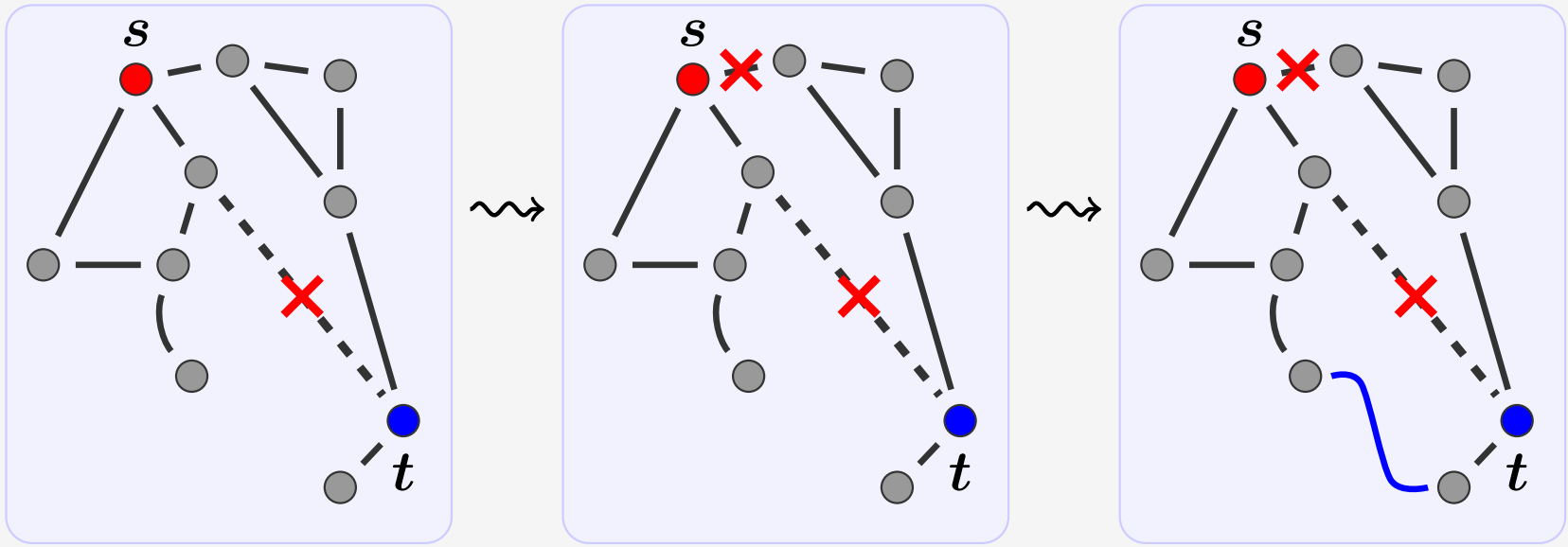

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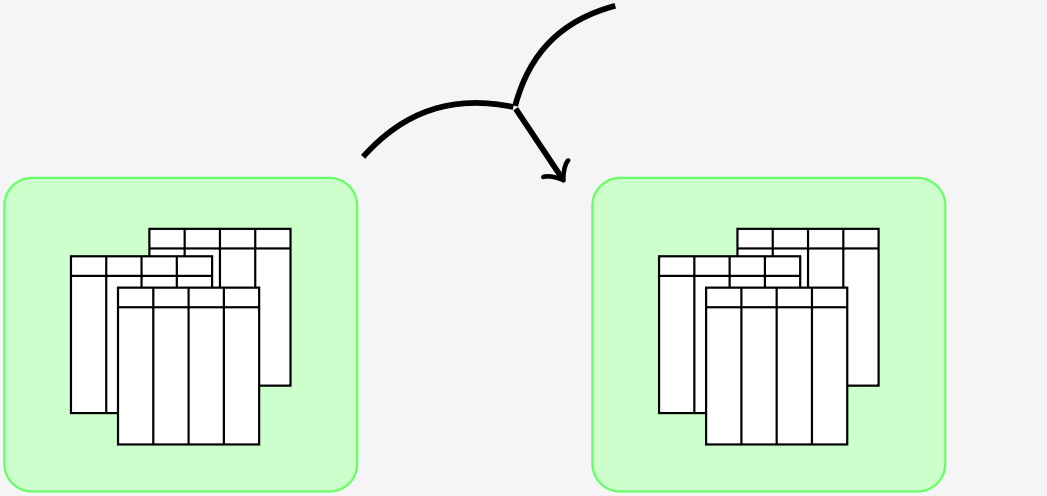

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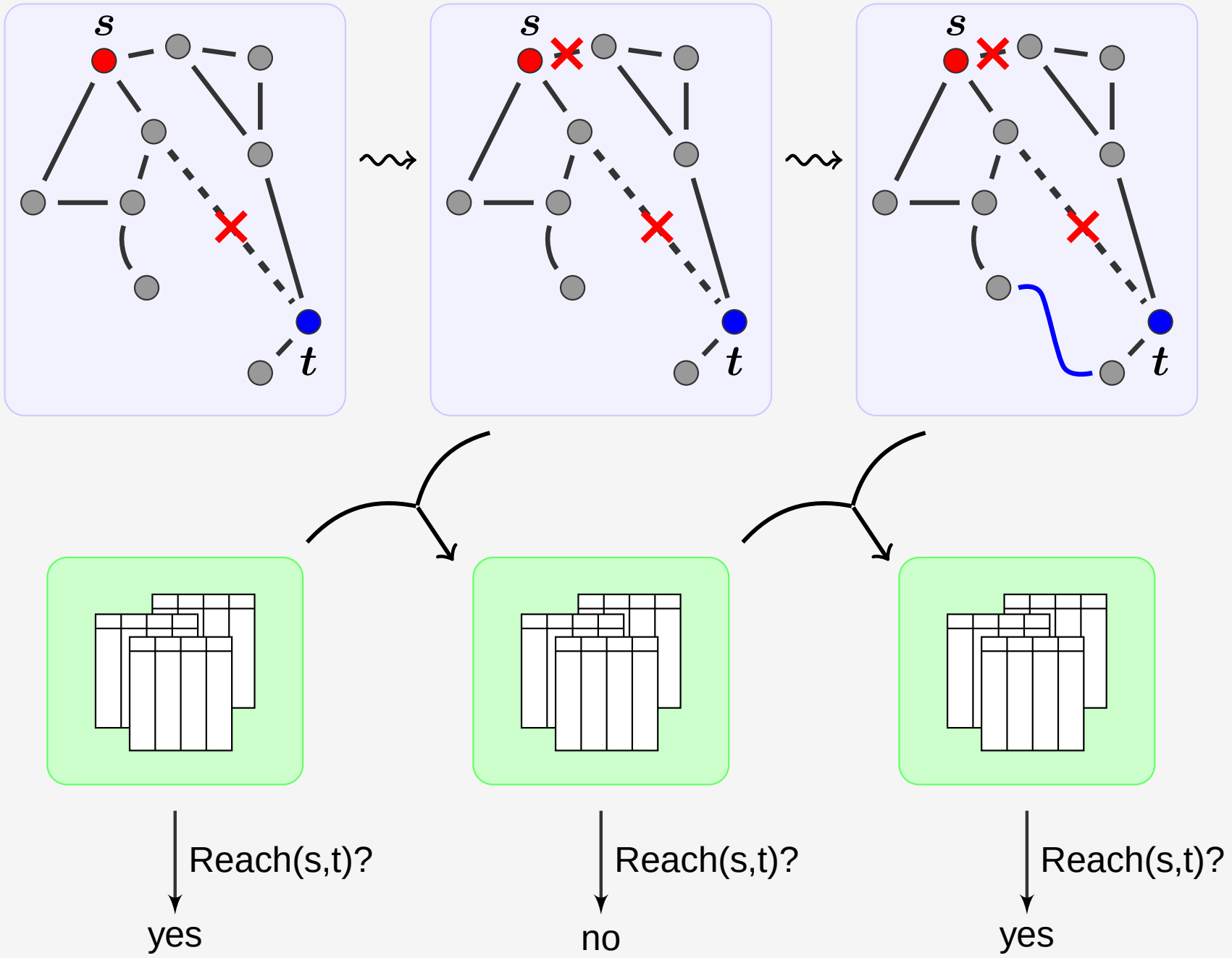

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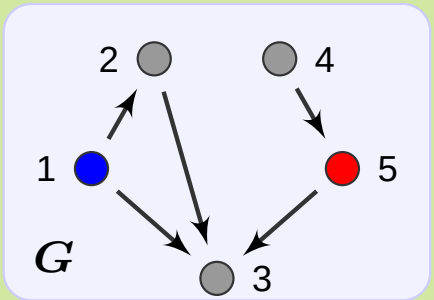

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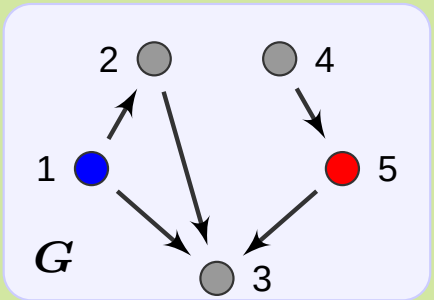

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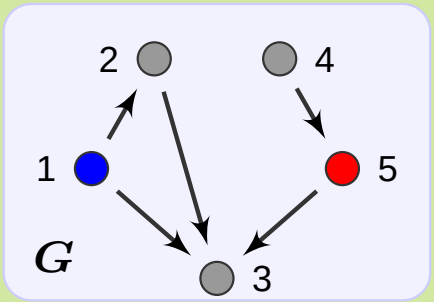

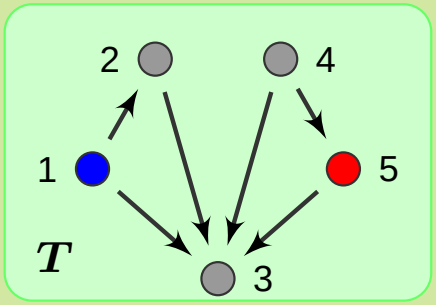

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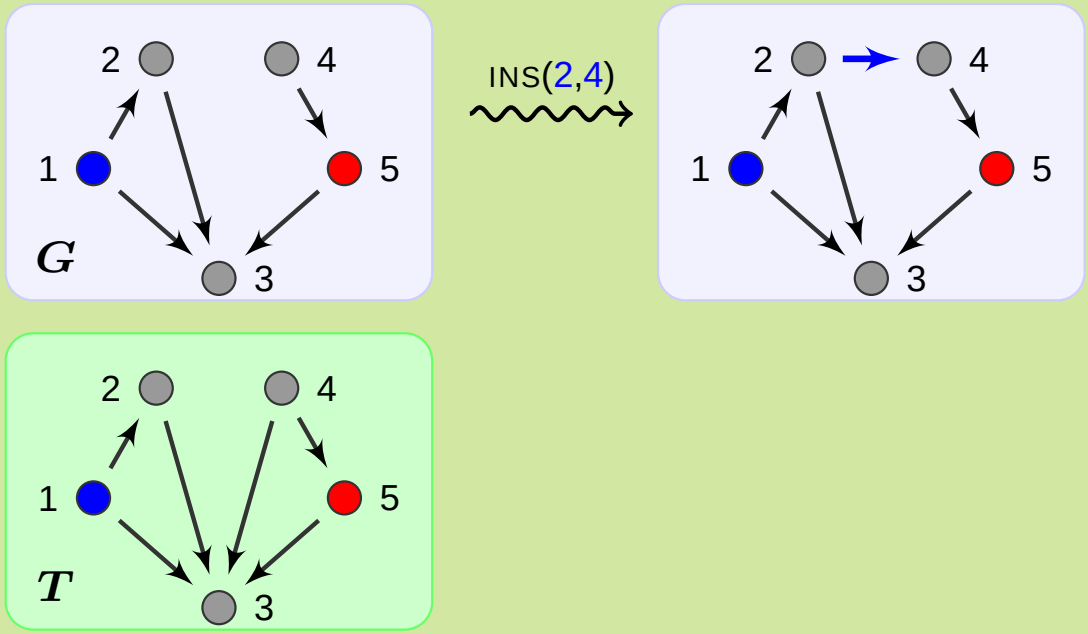

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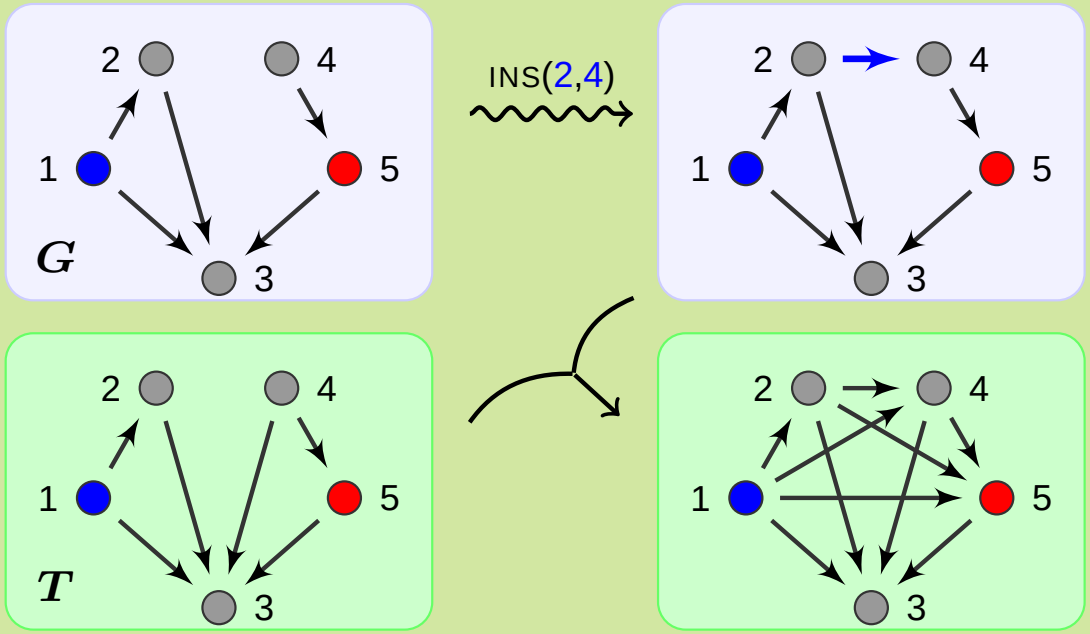

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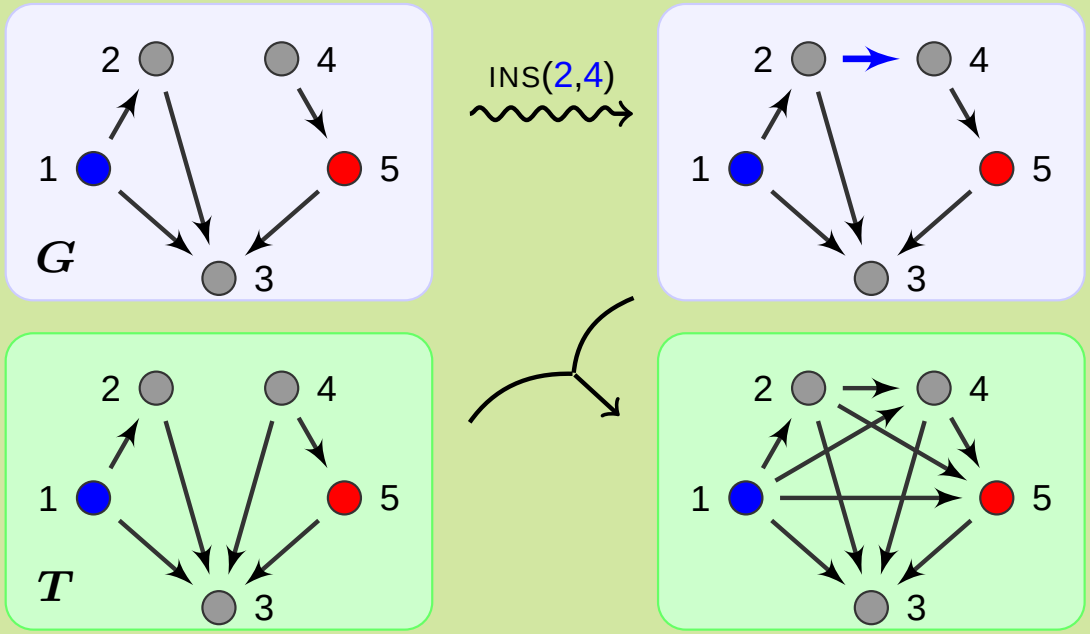

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- In this talk: Updates with fragments of first-order logic

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- Our results hold for **FO**-initialization as well

Previous Work: An Overview

DynFO (First-order update formulas)

- reachability in acyclic and undirected graphs [Patnaik et al. ’94]
- context-free languages [Gelade et al. ’09]

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  - (+ similar results)
- Question: What about other fragments of first-order logic?
  - Recall: (core) SQL $\approx$ first-order logic
- Goal of this talk: Typical restrictions of (core) SQL as update languages

Conjunctive Queries and their Extensions

- Most intensively studied query language: Conjunctive queries (**CQ**)

Example

$$\varphi(x) = \exists y_1 \exists y_2 \big(T(x, y_1) \wedge T(y_1, y_2)\big)$$

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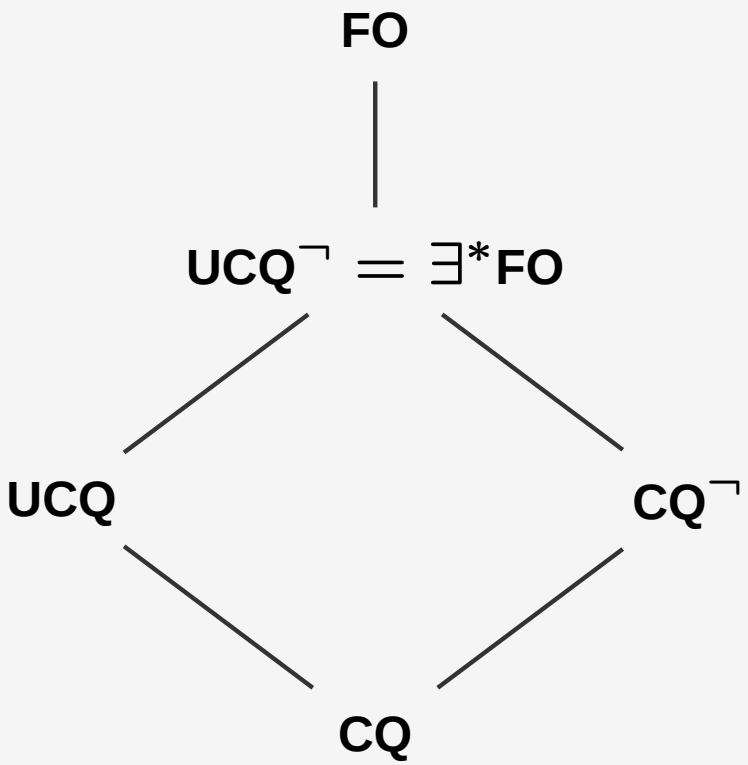

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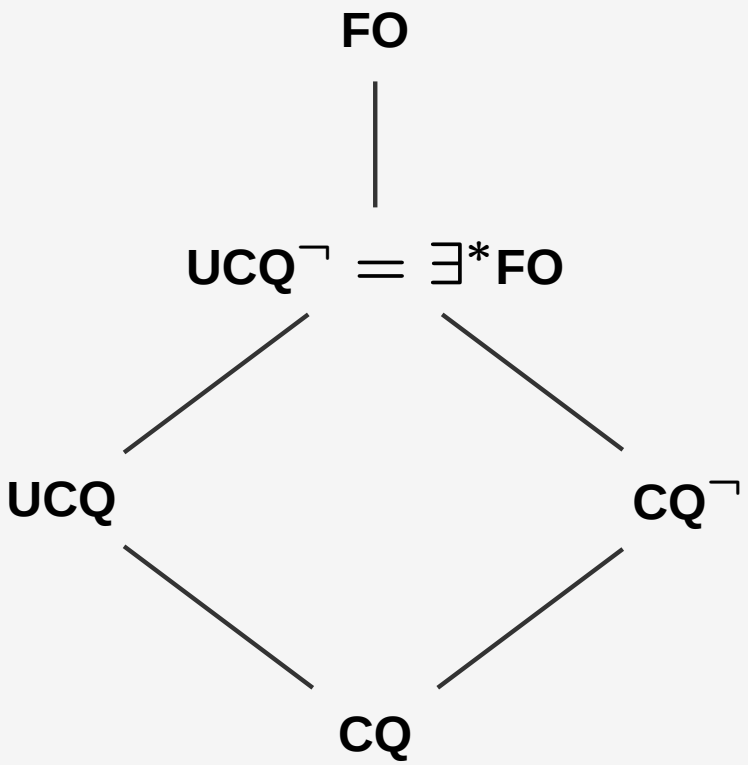

- In this talk: Conjunctive queries (and extensions thereof) as update language
  - How powerful are they in the dynamic setting?
  - How do the dynamic variants relate to each other?

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The Framework of Patnaik and Immerman

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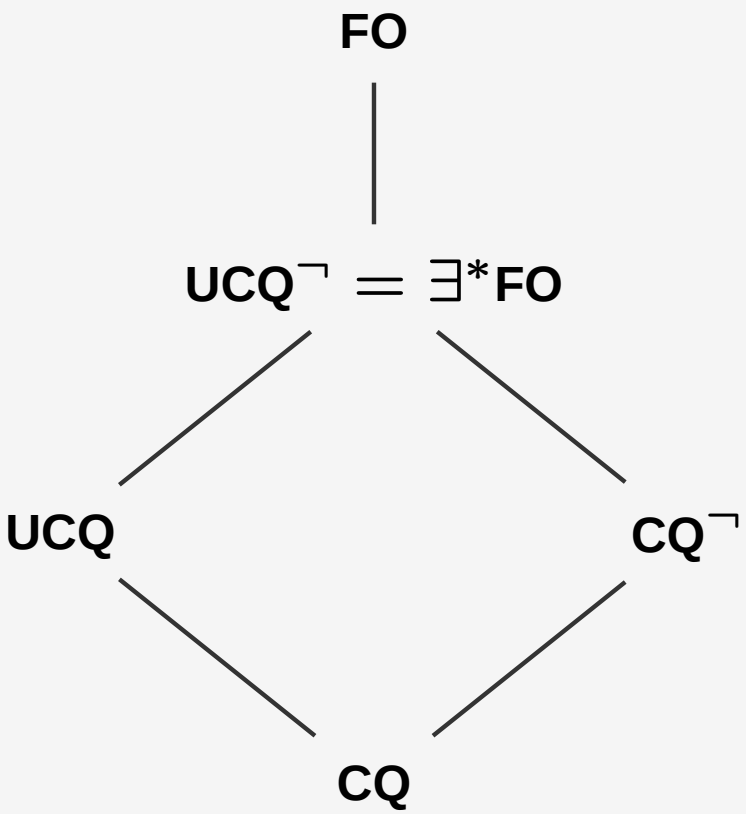
How do Variants of Dynamic Conjunctive Queries relate?

- Conjunctive queries in the static world



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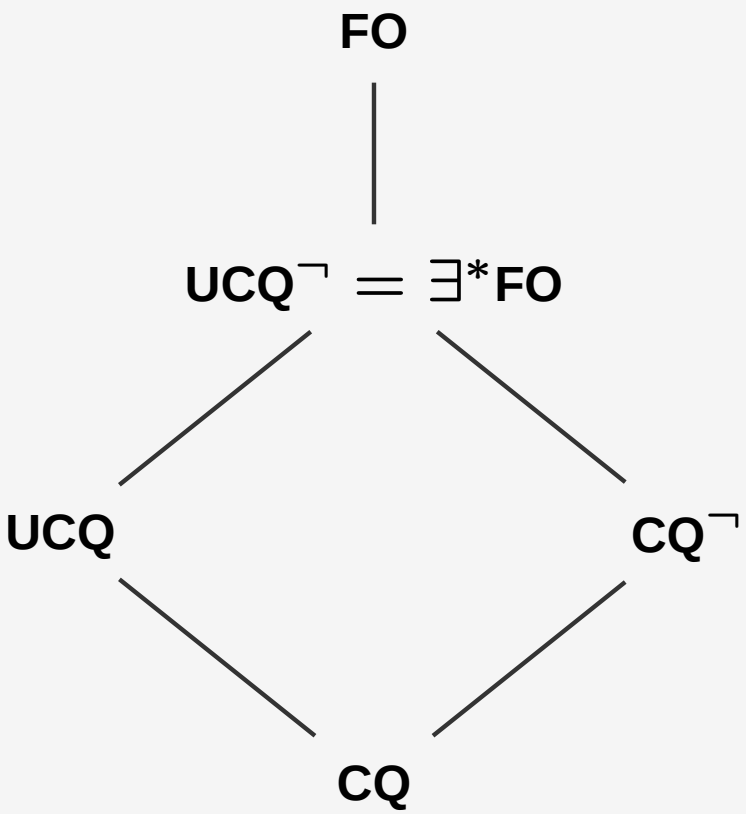

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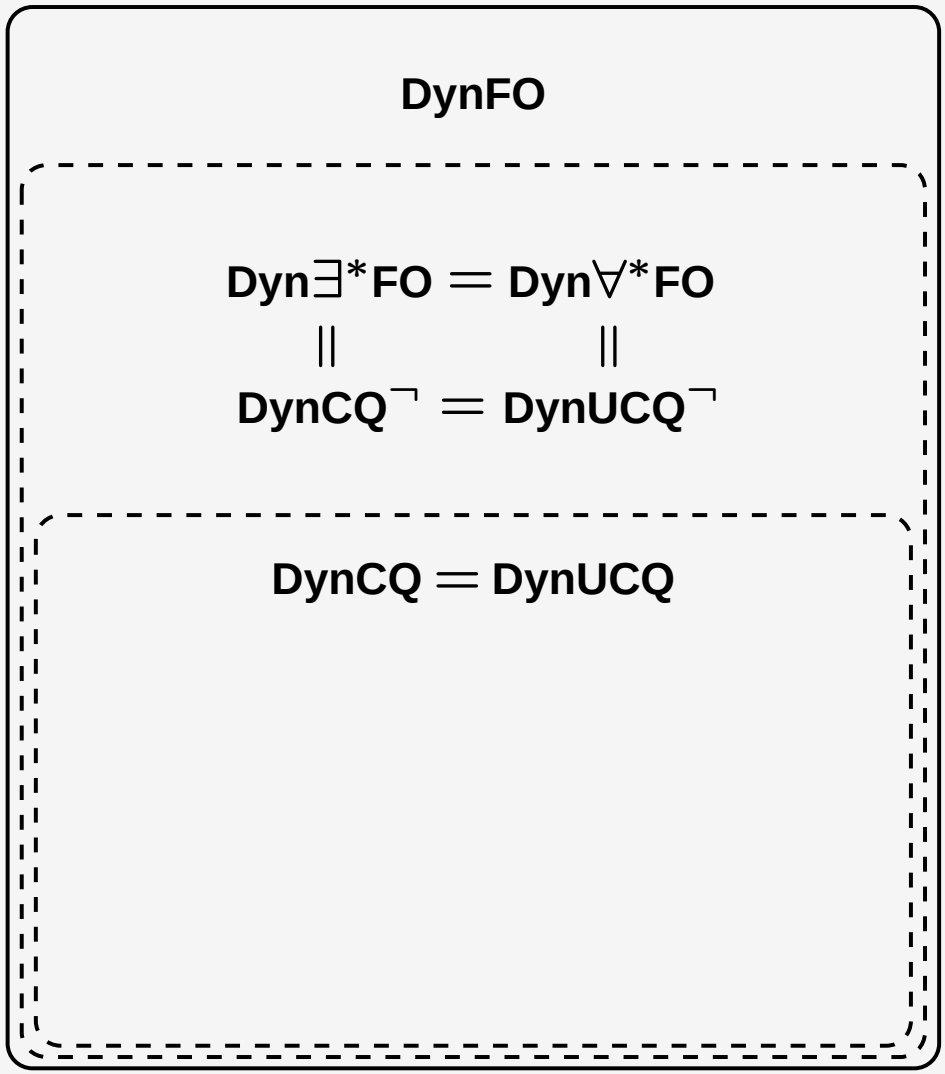

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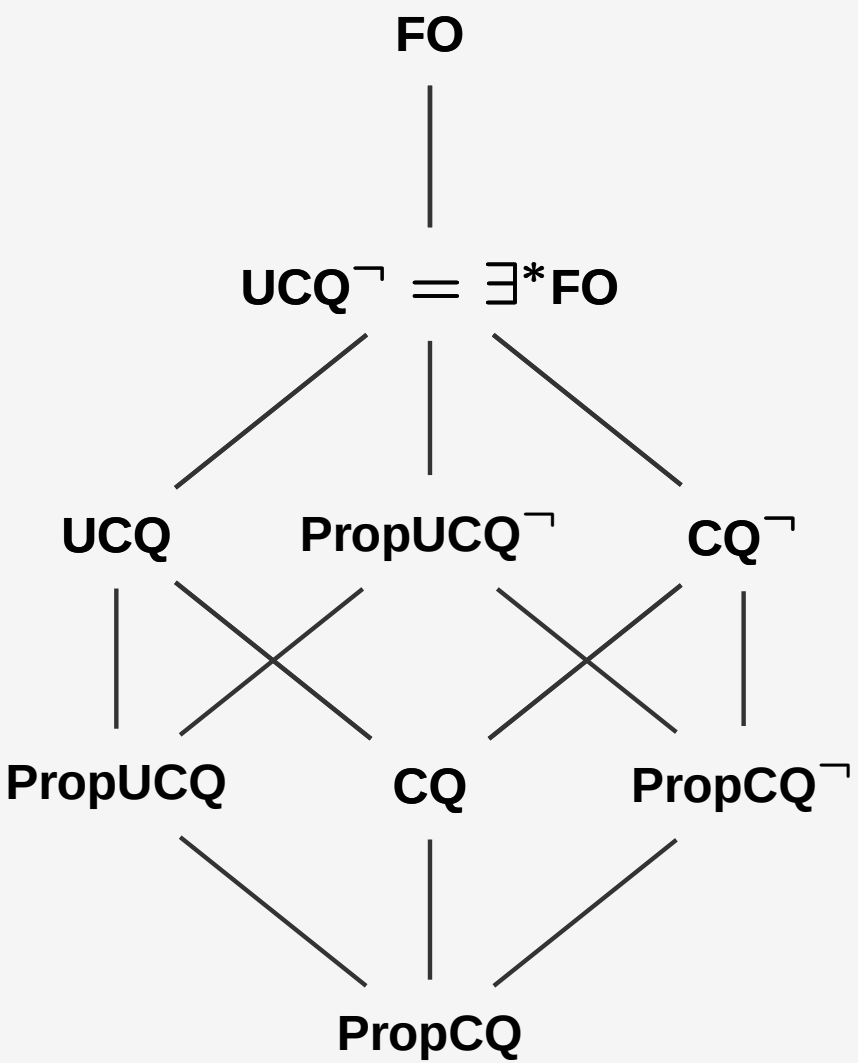

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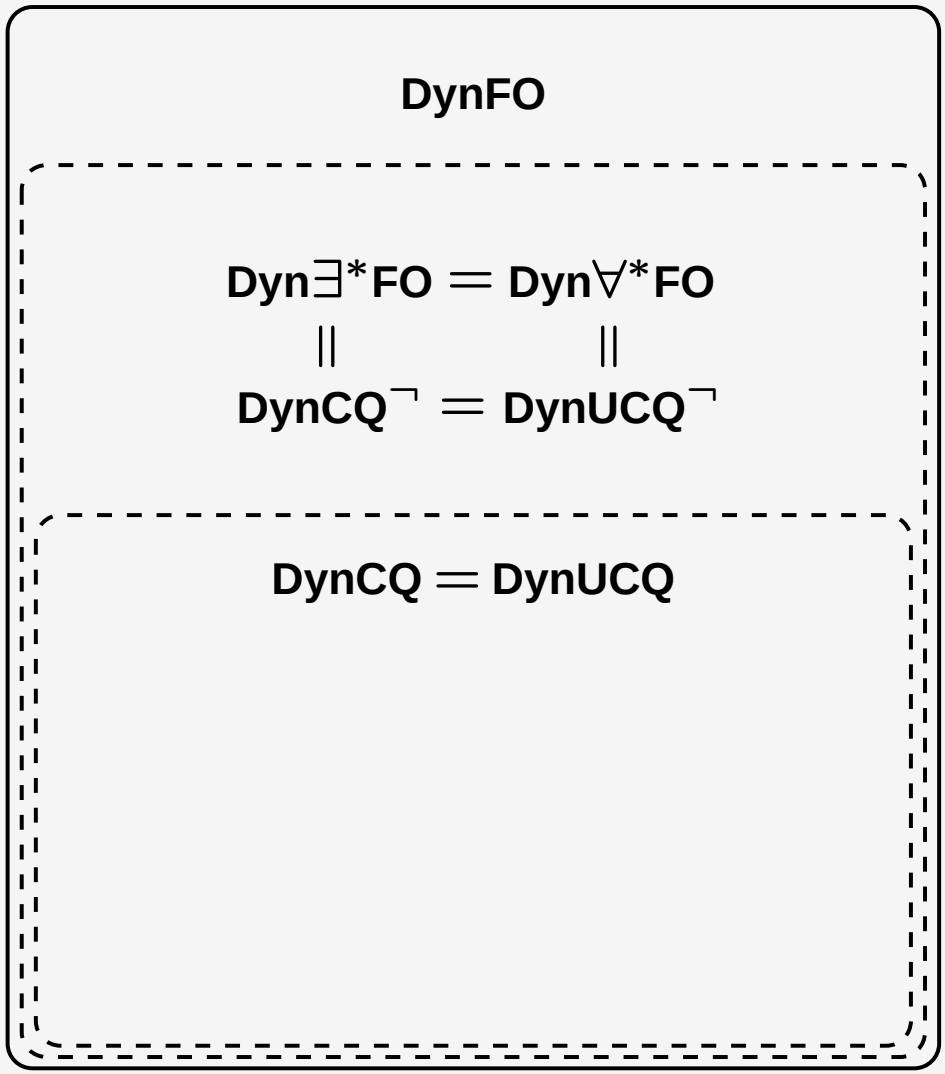

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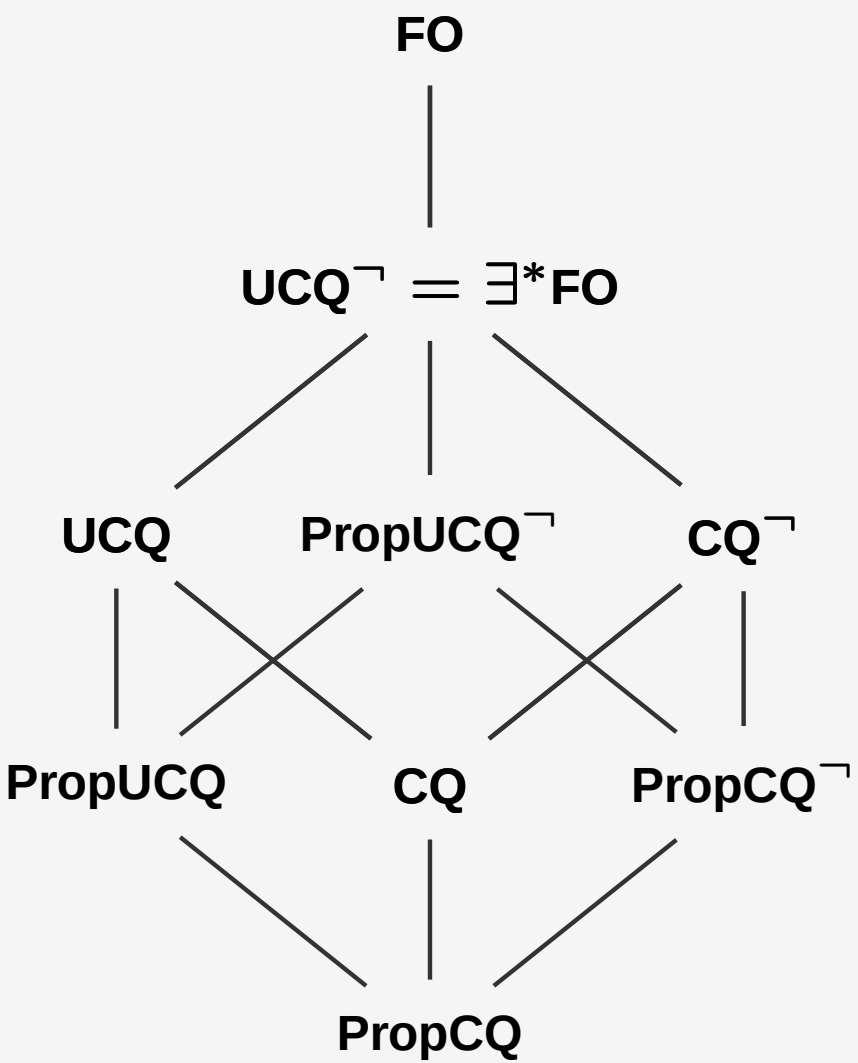

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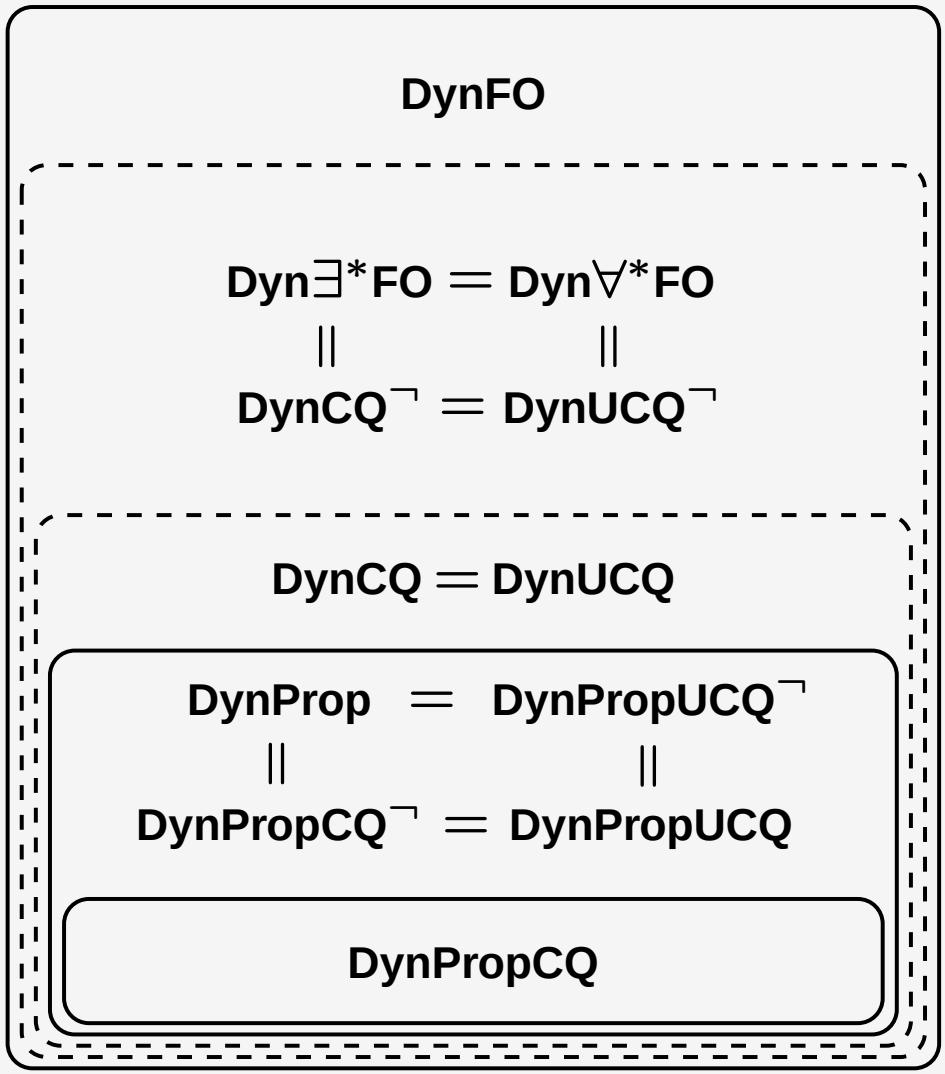

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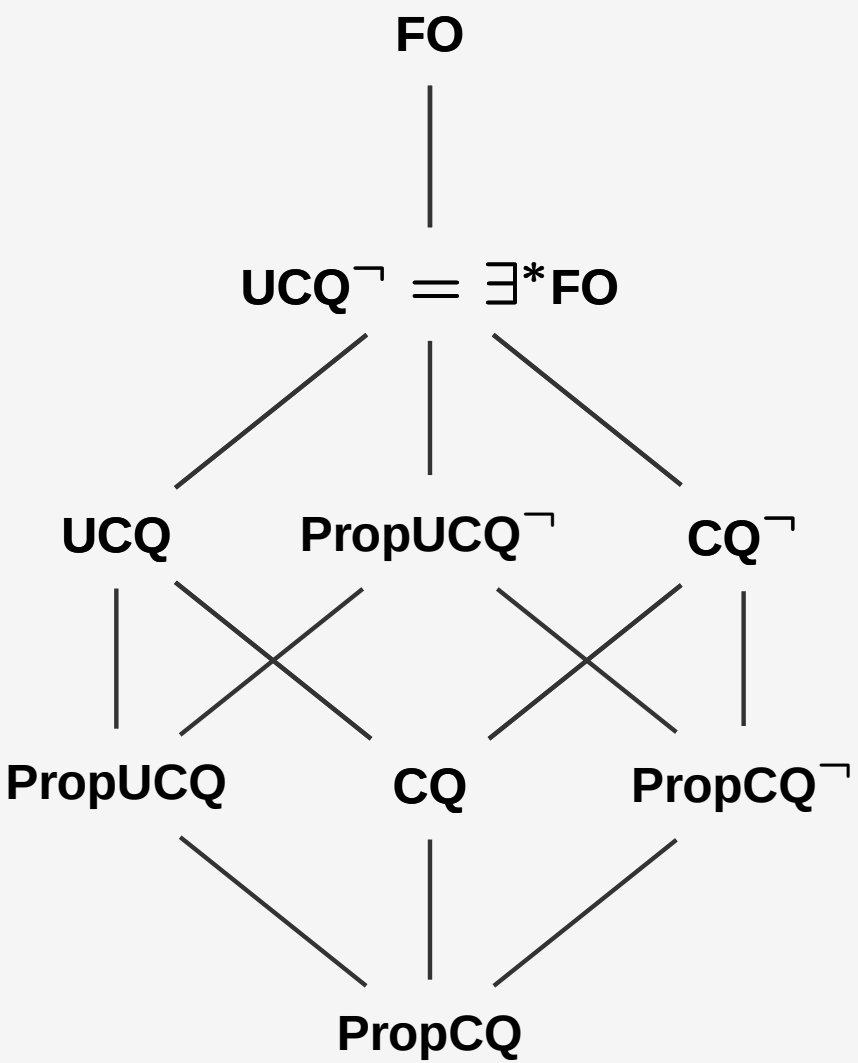

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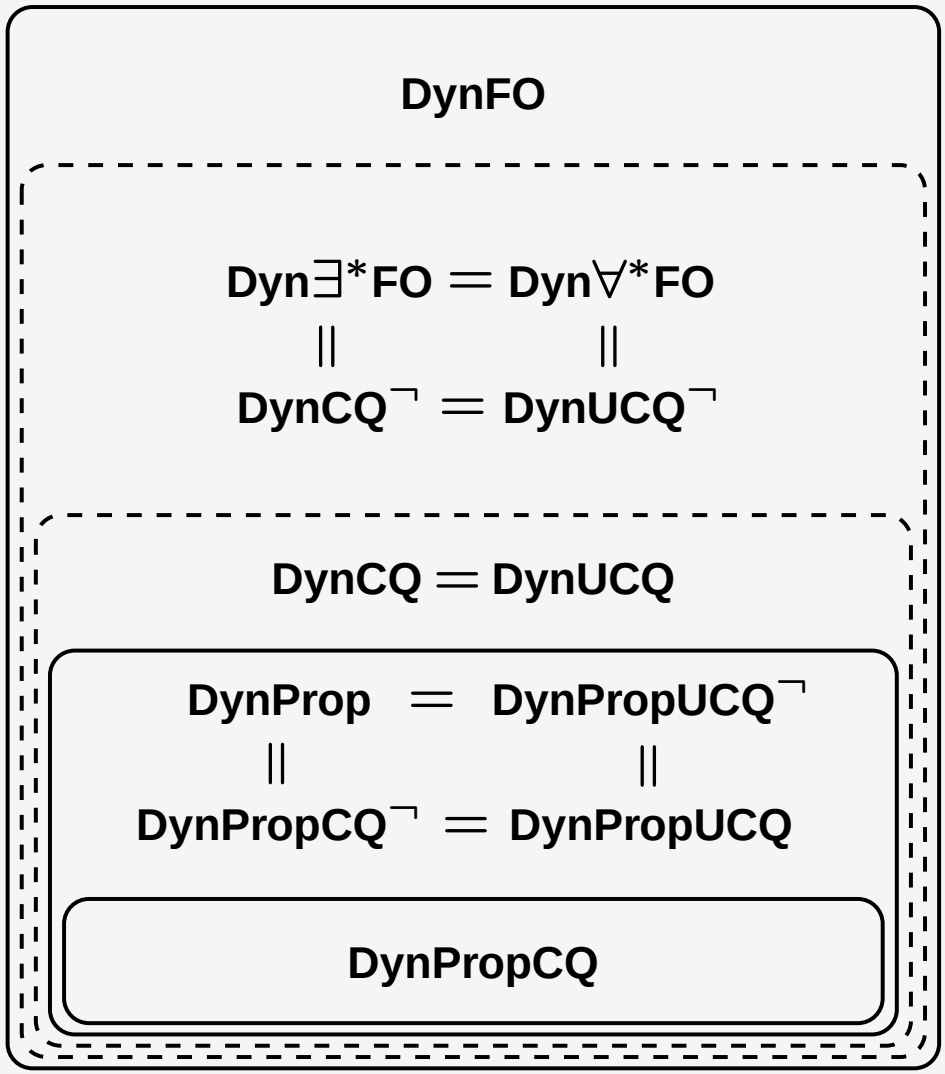

- Solid lines are strict separations

A Glimpse at the Proofs: How to remove Unions

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where $P(x,y,u,v,z_1,\ldots,z_6)$ is (**FO**-)initialized by the result of

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An Alternative Semantics

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  - Auxiliary data defined from scratch

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→ Questions:

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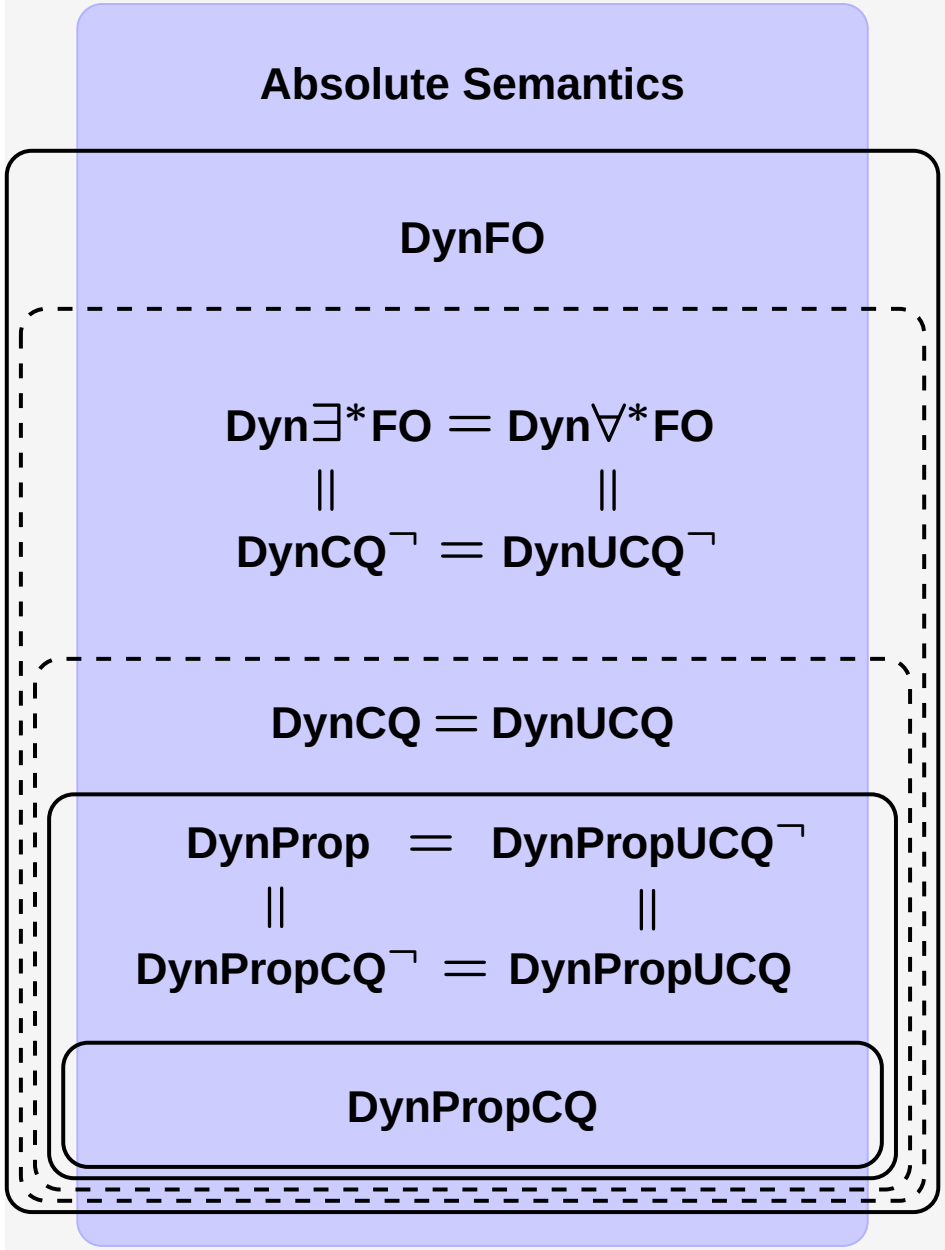
  - How do the semantics compare to each other?

Absolute Semantics vs. Δ-Semantics



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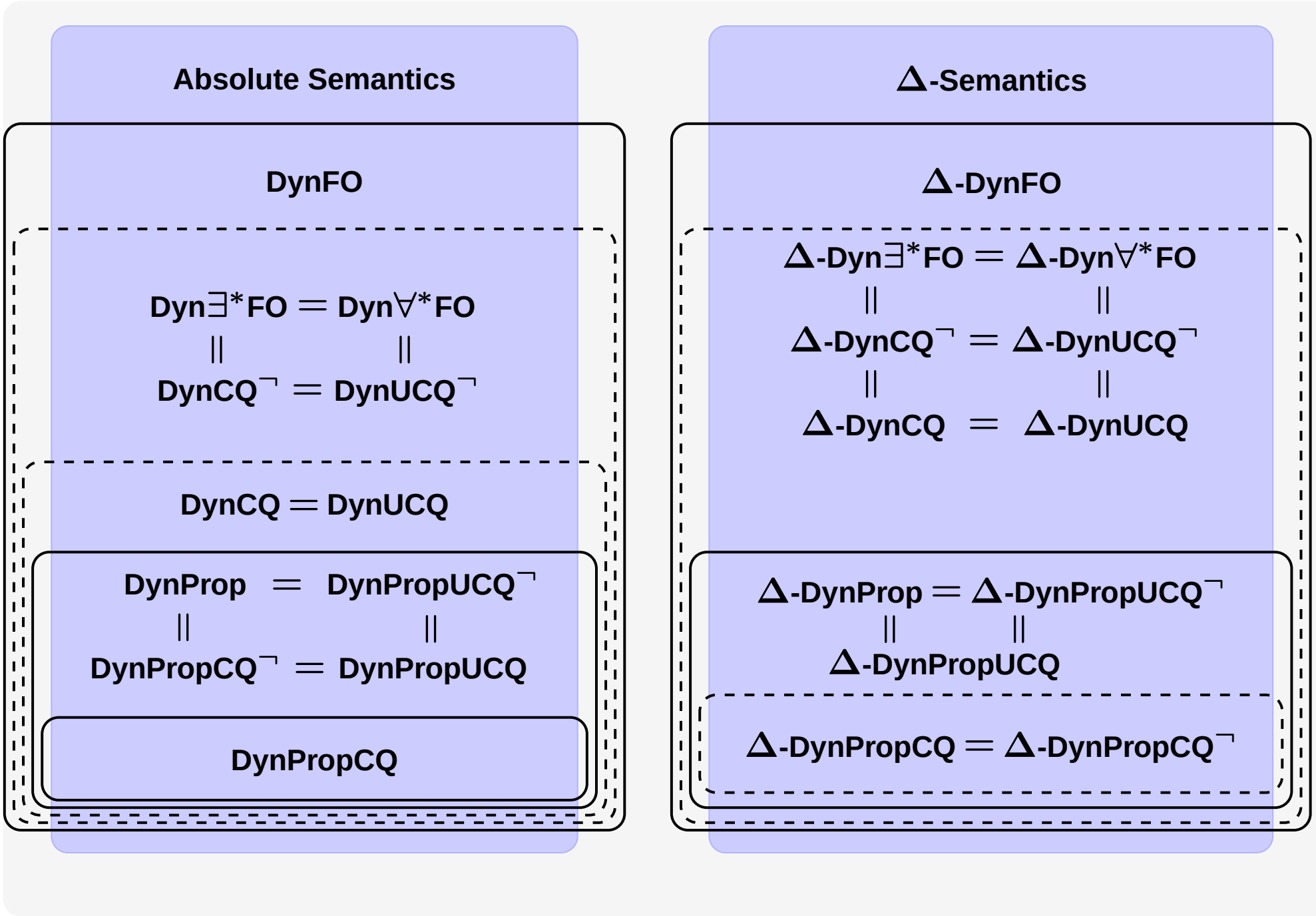

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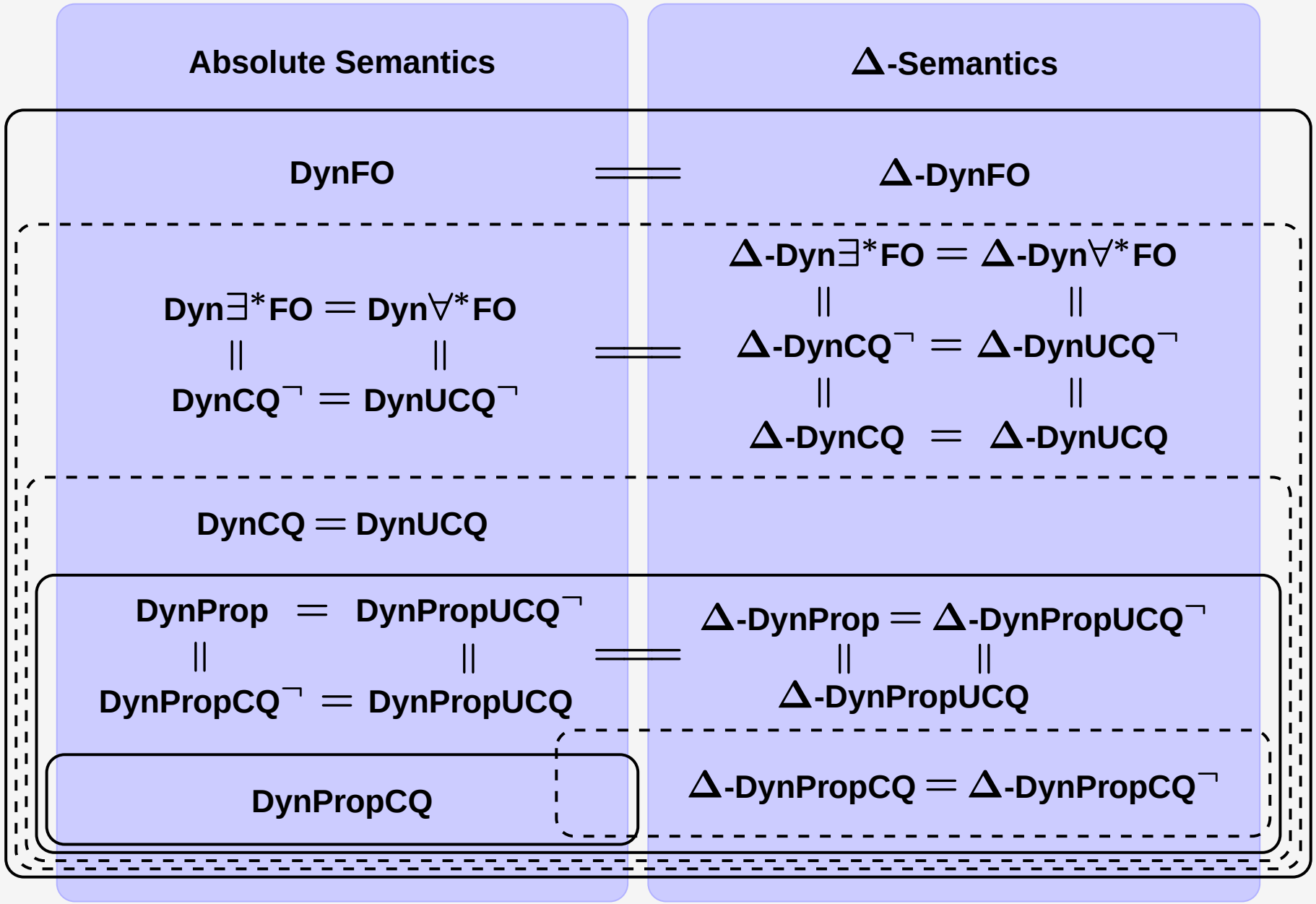

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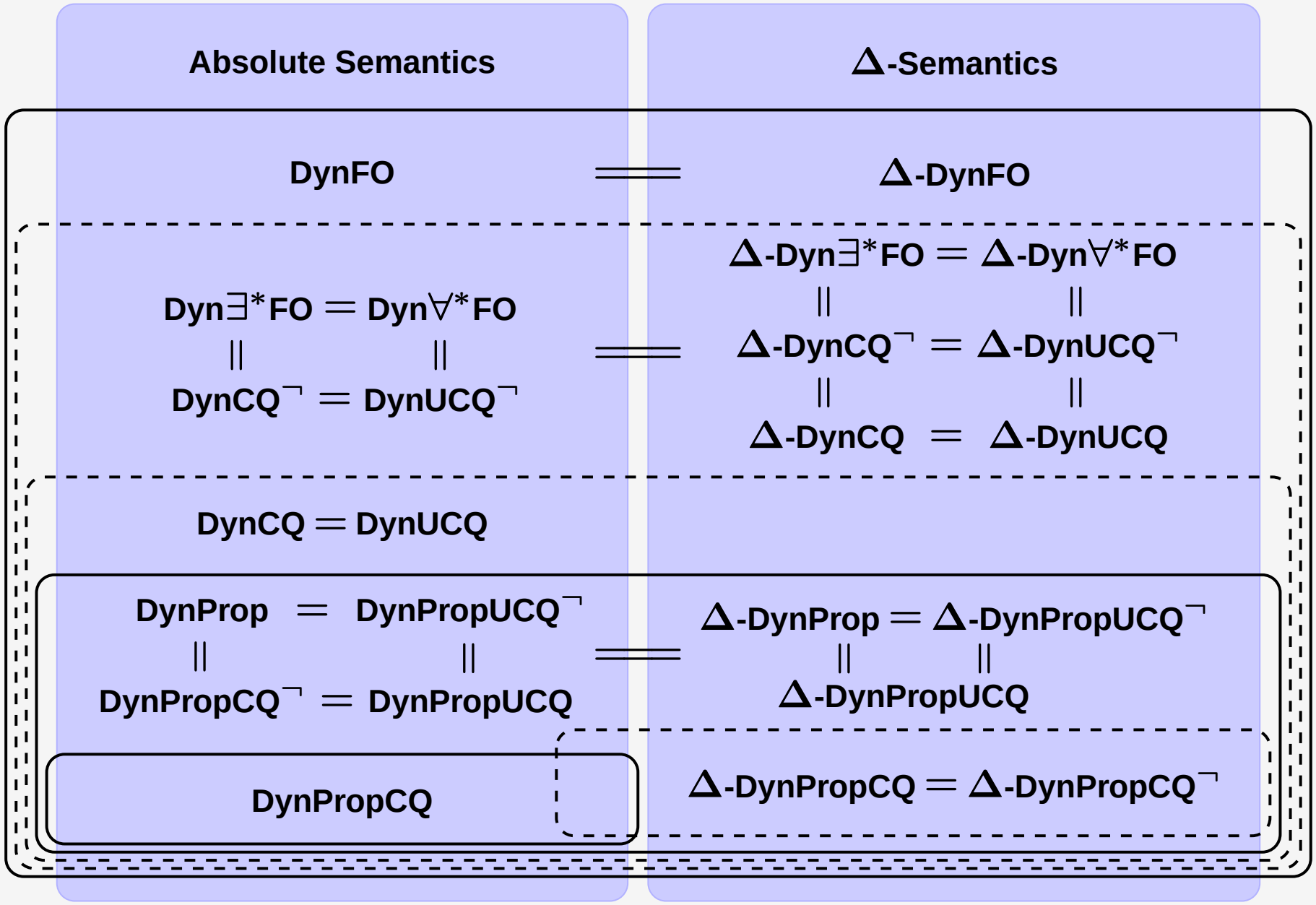

- Hence: Every first-order query is maintainable in **Δ-DynCQ**.

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Conclusion and Future Work

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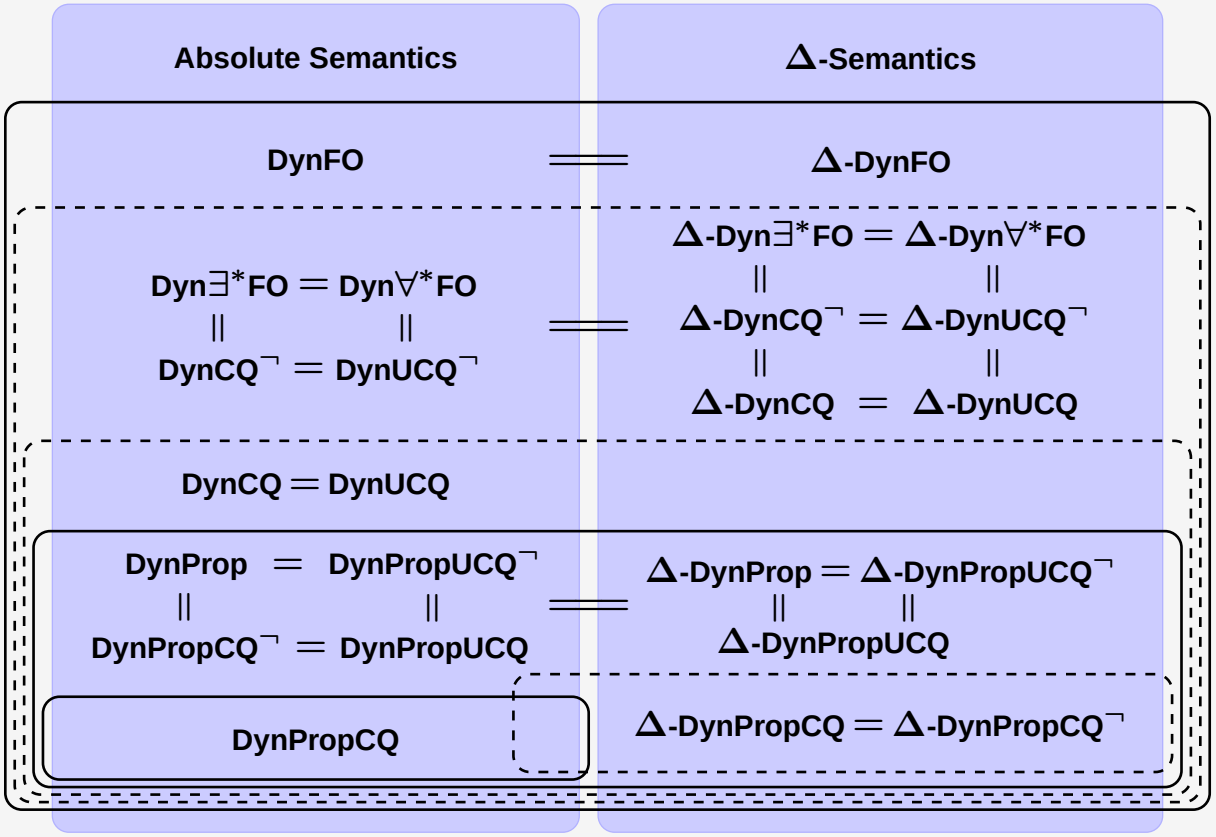
  - **FO $\subsetneq$ DynCQ$^\neg$**
  - Variants of dynamic conjunctive queries:



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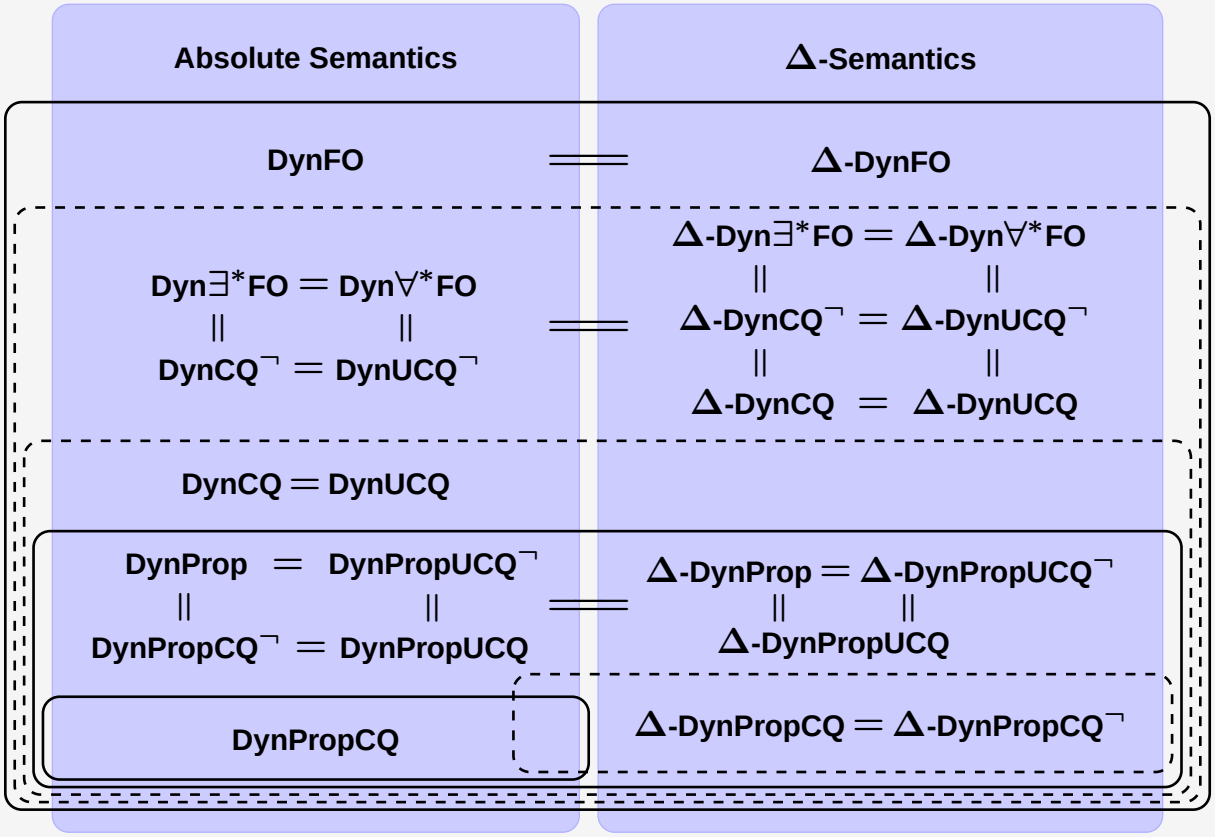

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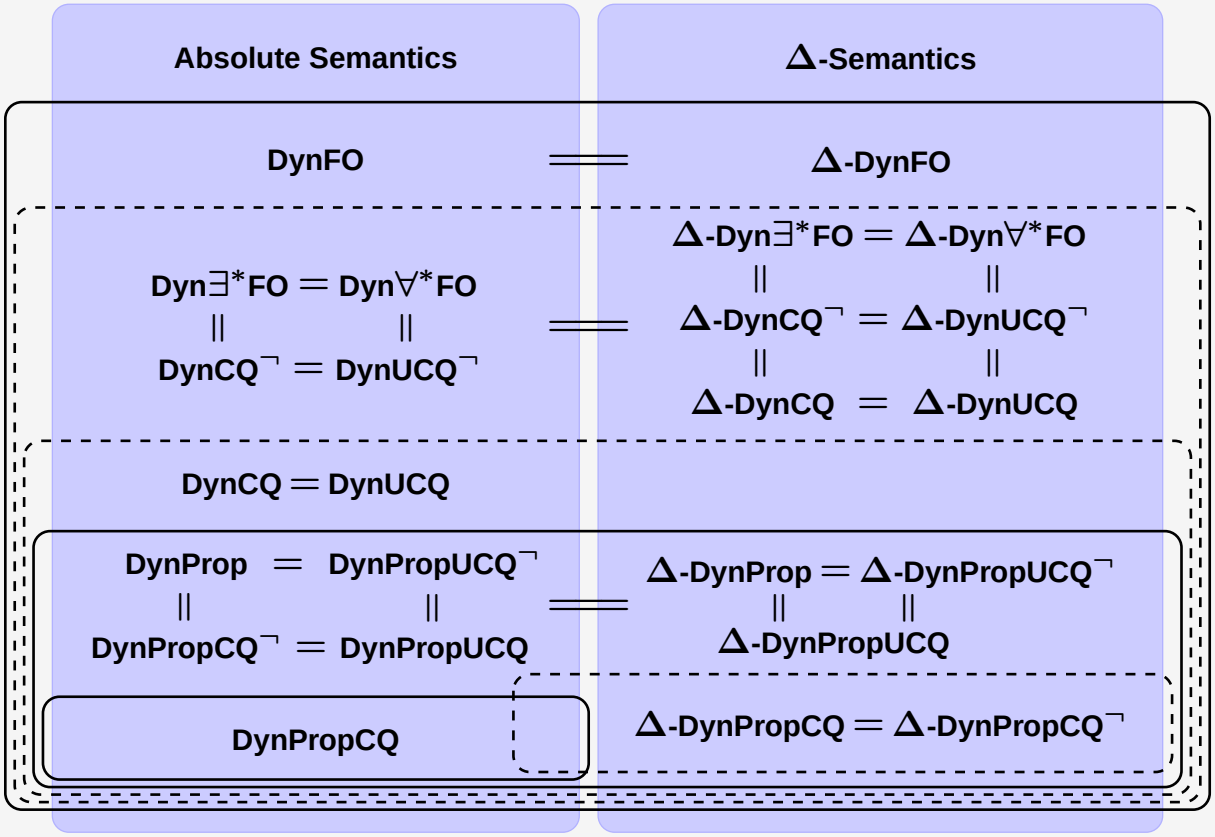

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Thank You!

And thanks to the DFG for supporting this work.